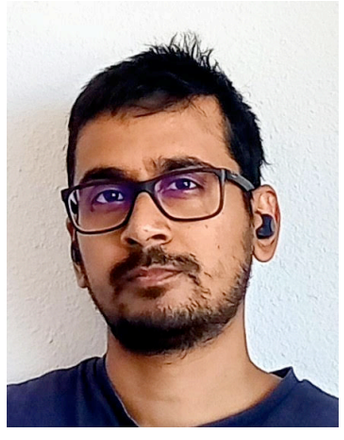
Reachability in extensions of Petri nets

FCT 2025, Wrocław, 2025-09-15

Wojciech Czerwiński

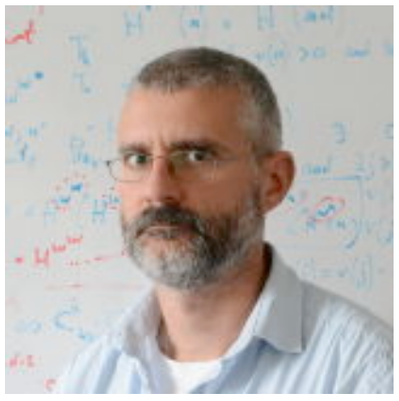


Arka Ghosh

Roland Guttenberg

Ismael Jecker

Łukasz Kamiński



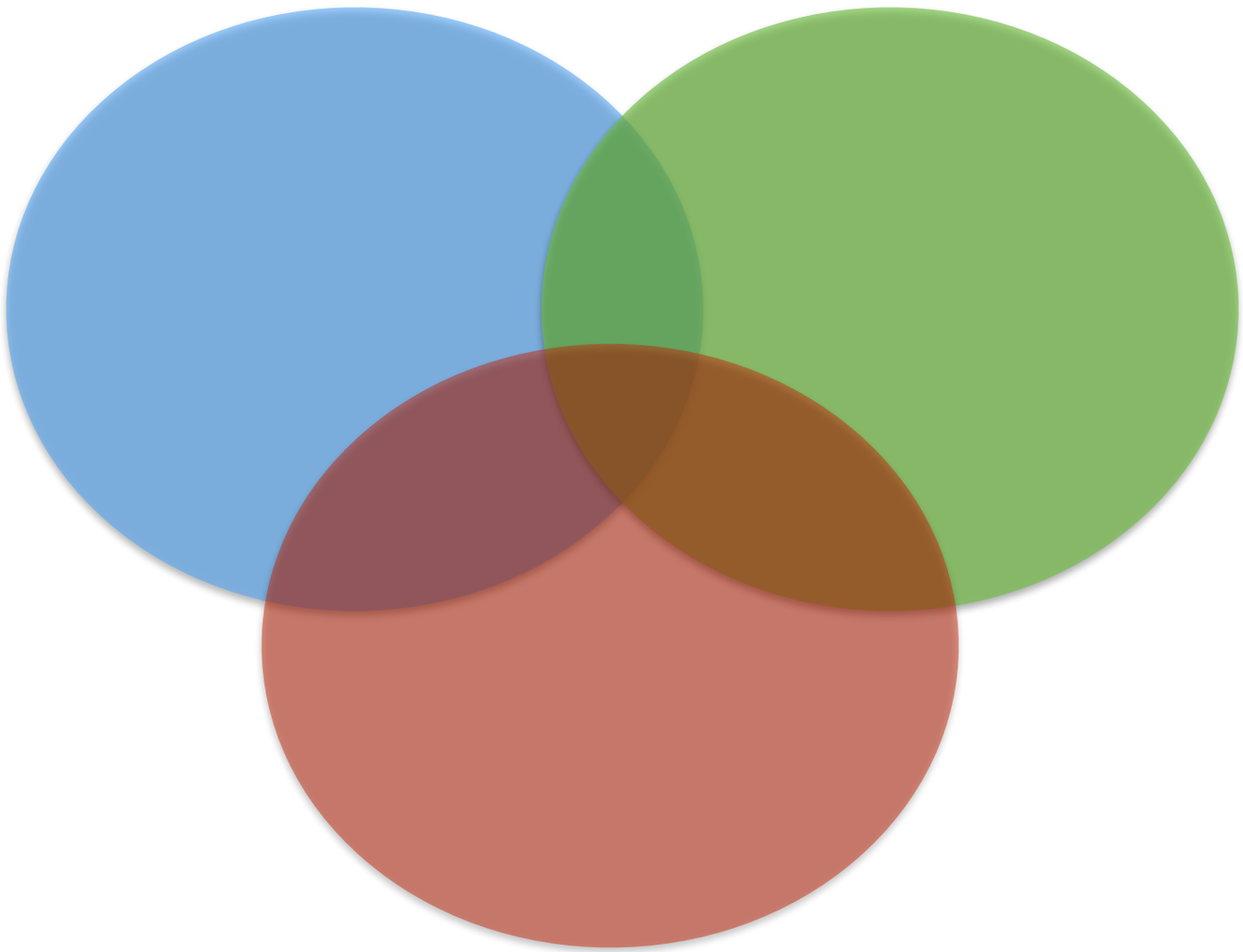
Ranko Lazic

Jerome Leroux

Filip Mazowiecki

Łukasz Orlikowski

Sylvain Schmitz

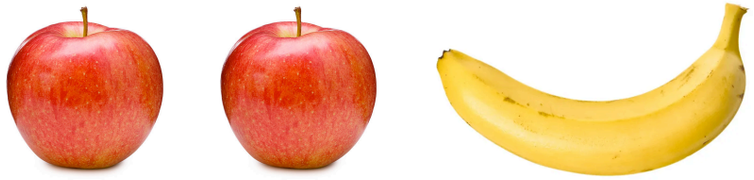


I. Reachability in Petri nets

II. Extensions of Petri nets

Kindergarten fruit-trading

Kindergarten fruit-trading

Alice has  but likes 

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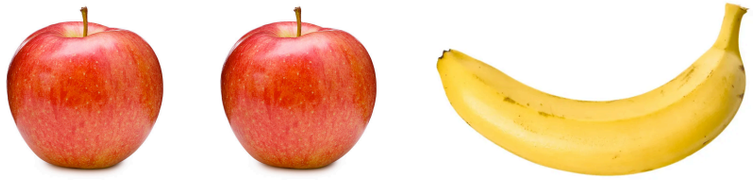
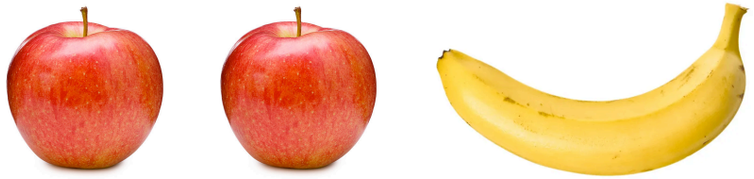
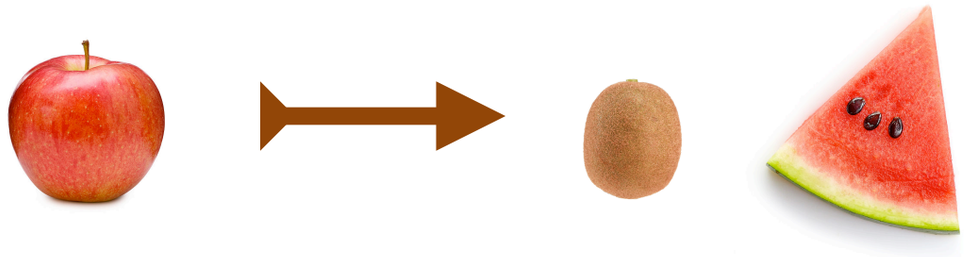
Alice has  but likes 

Kindergarten fruit-trading rules:

Kindergarten fruit-trading

Alice has  but likes 

Kindergarten fruit-trading rules:



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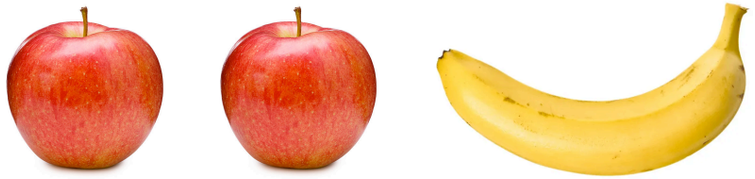
Alice has  but likes 

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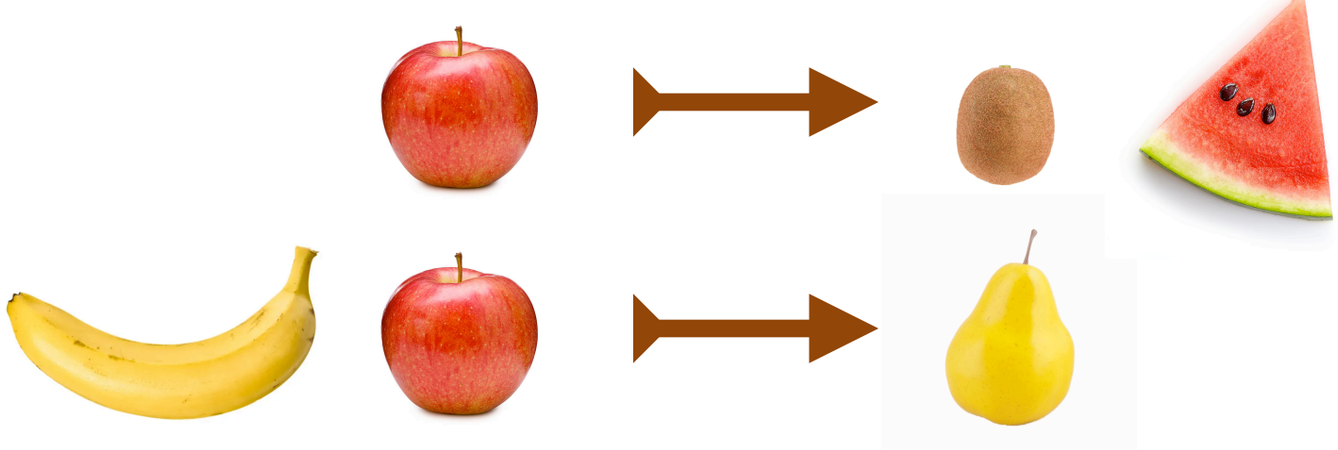

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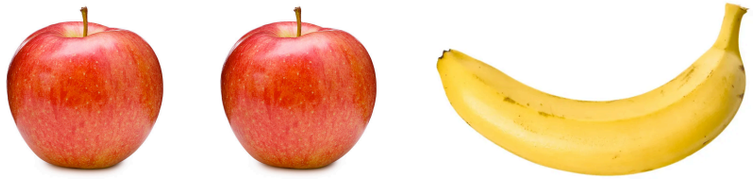
Alice has  but likes 

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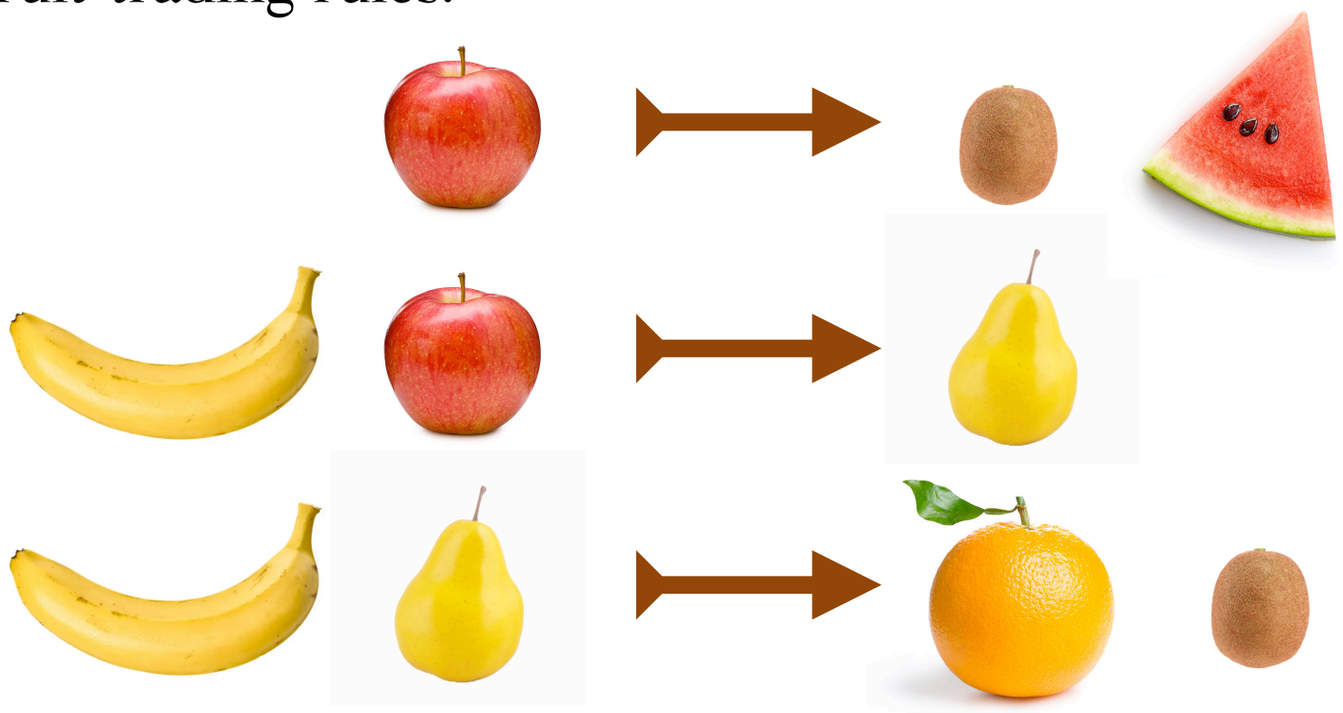

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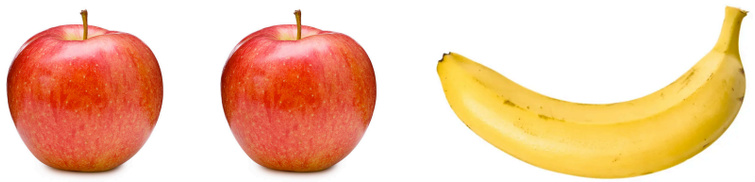
Alice has  but likes 

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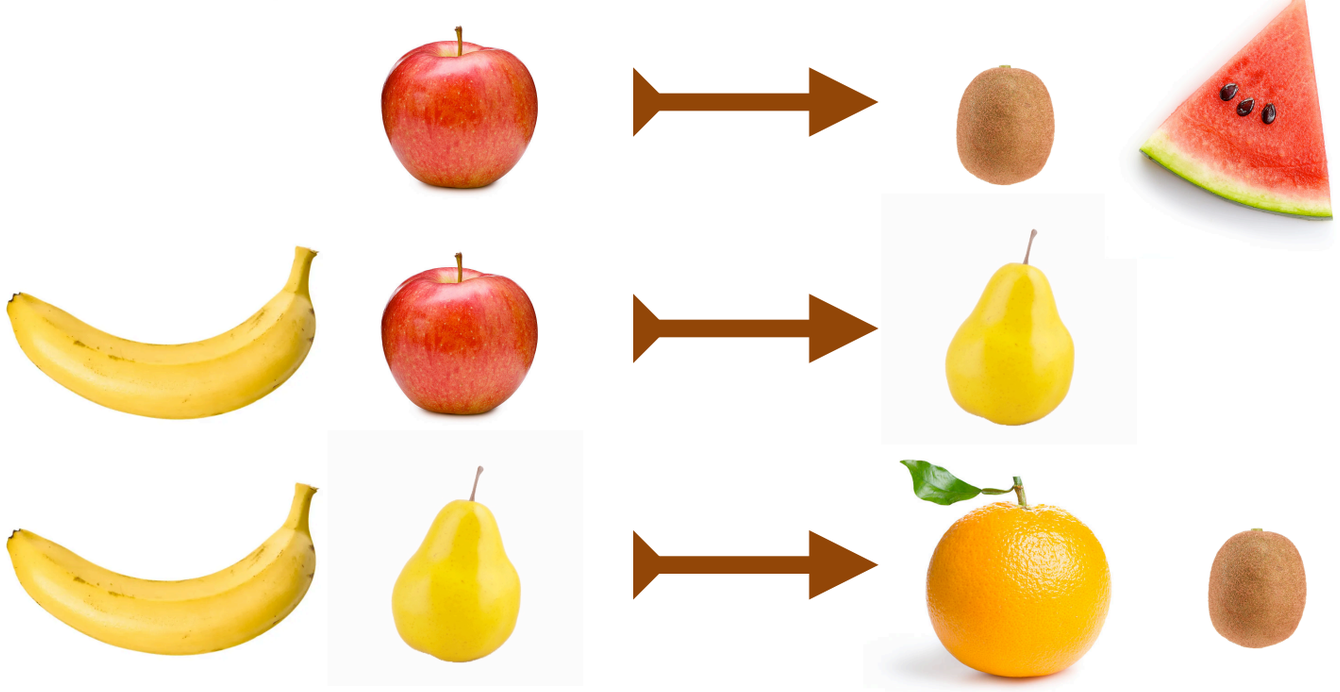

Decision problem:

 *  ?

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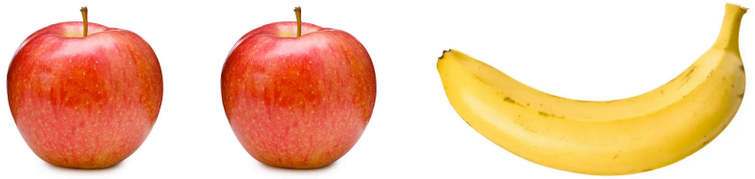
Alice has  but likes 

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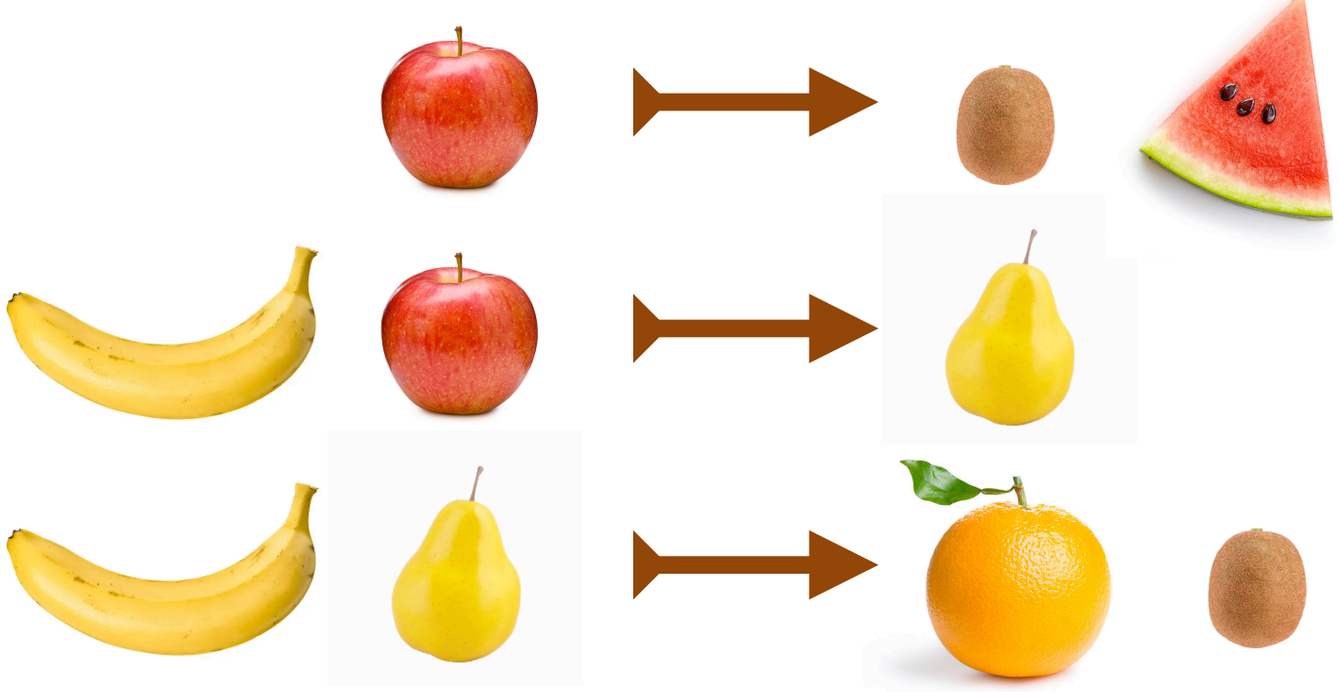

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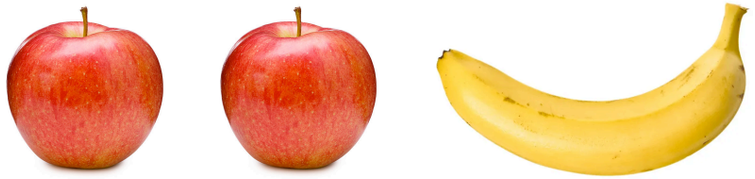
Alice has  but likes 

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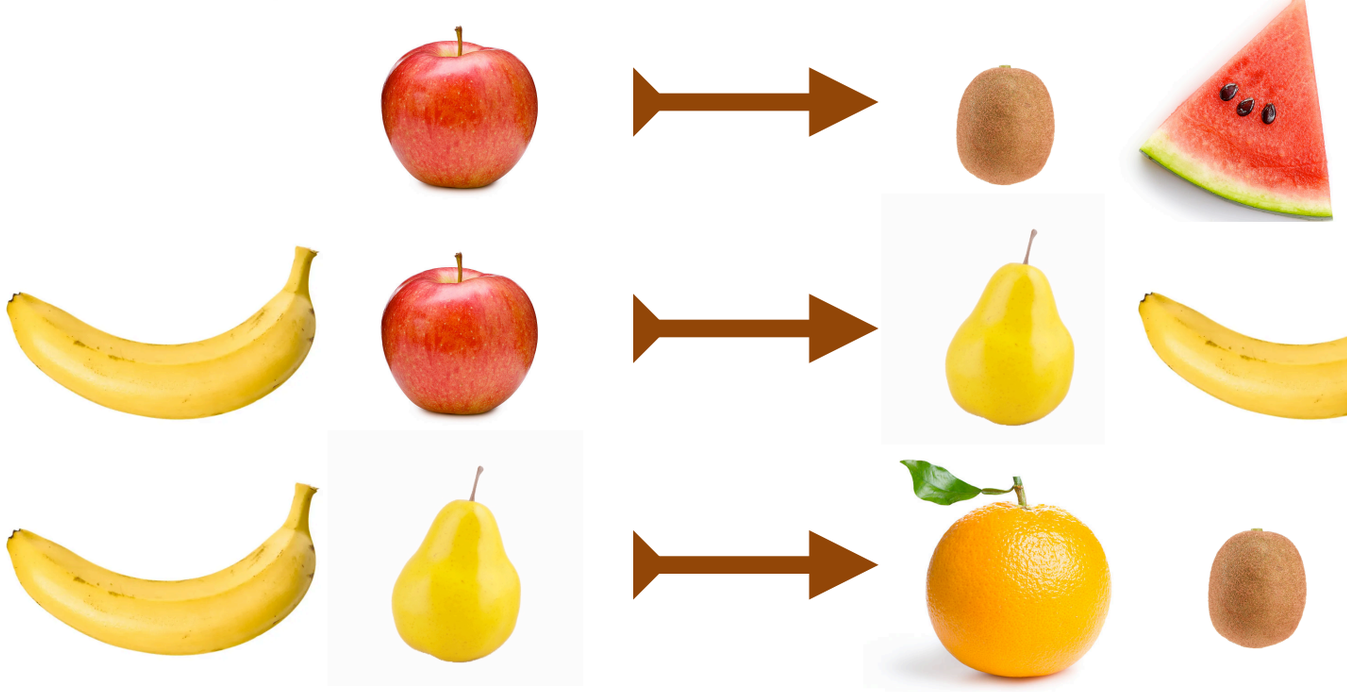

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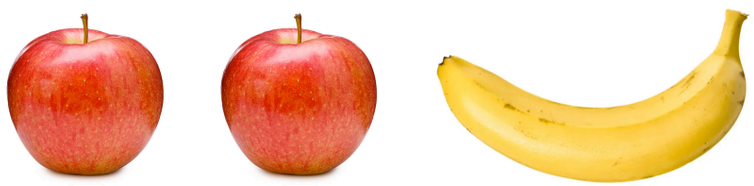
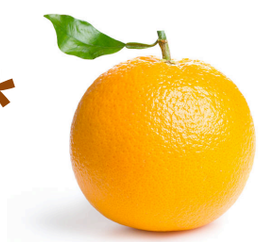
 *   ?

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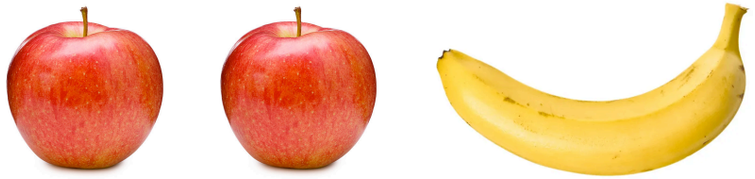
Alice has  but likes 

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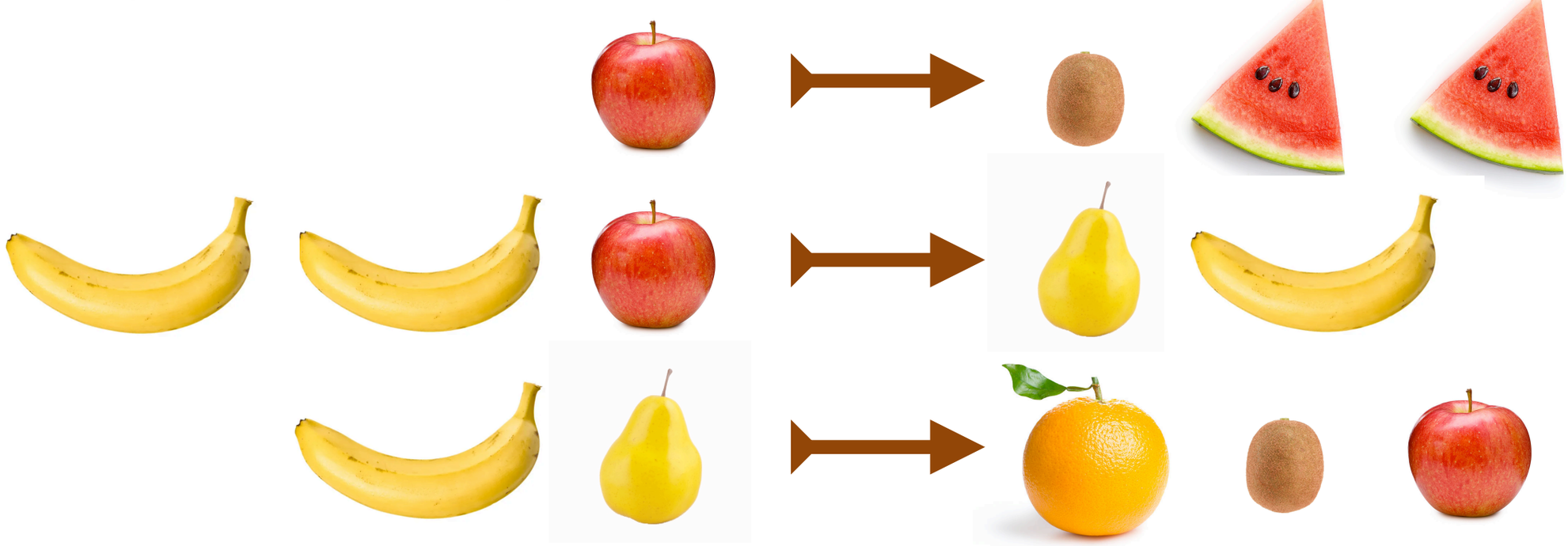

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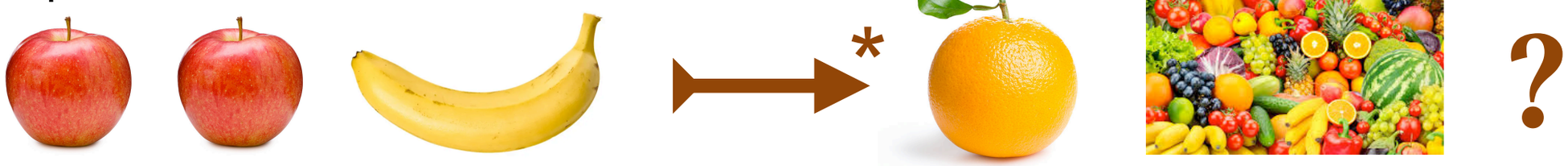
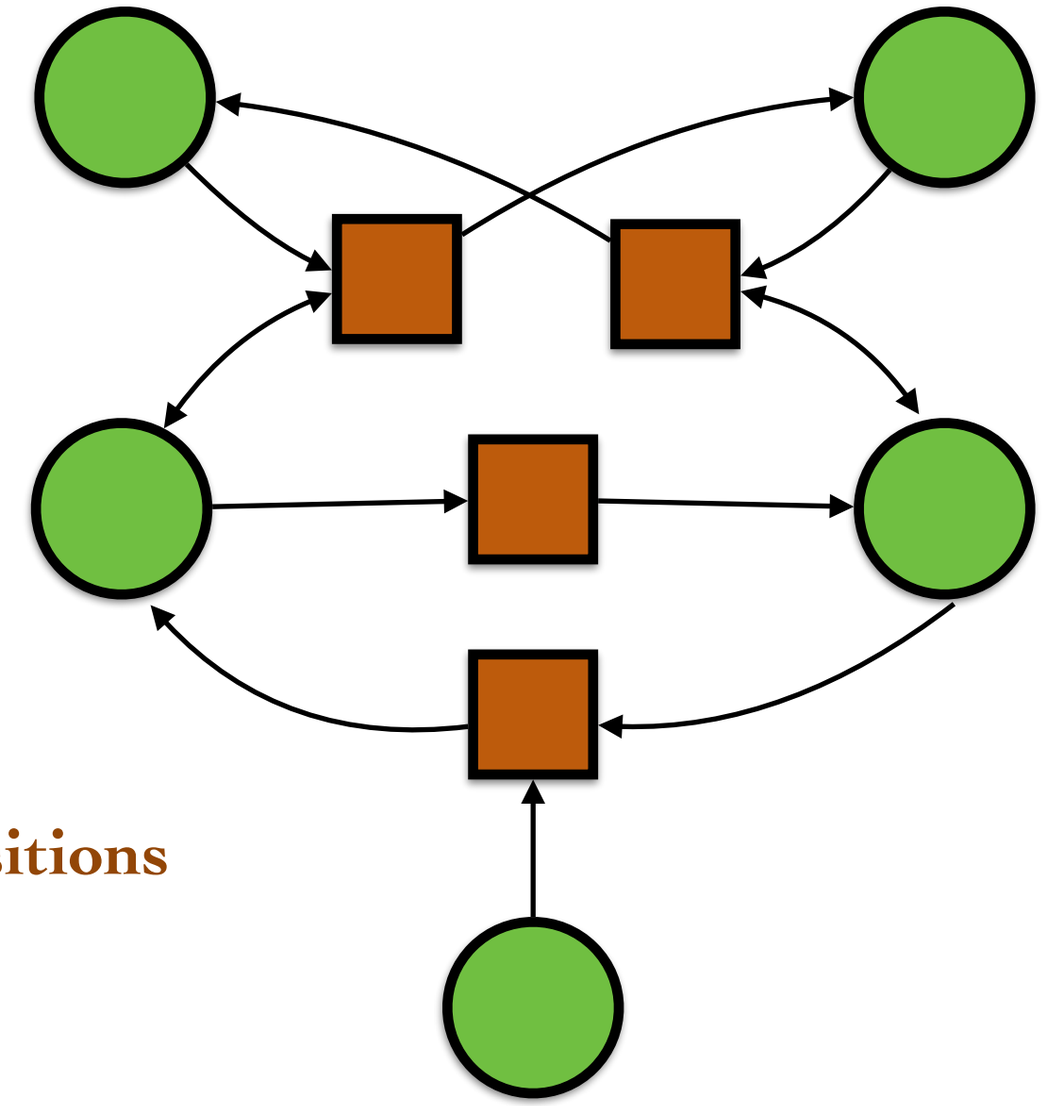


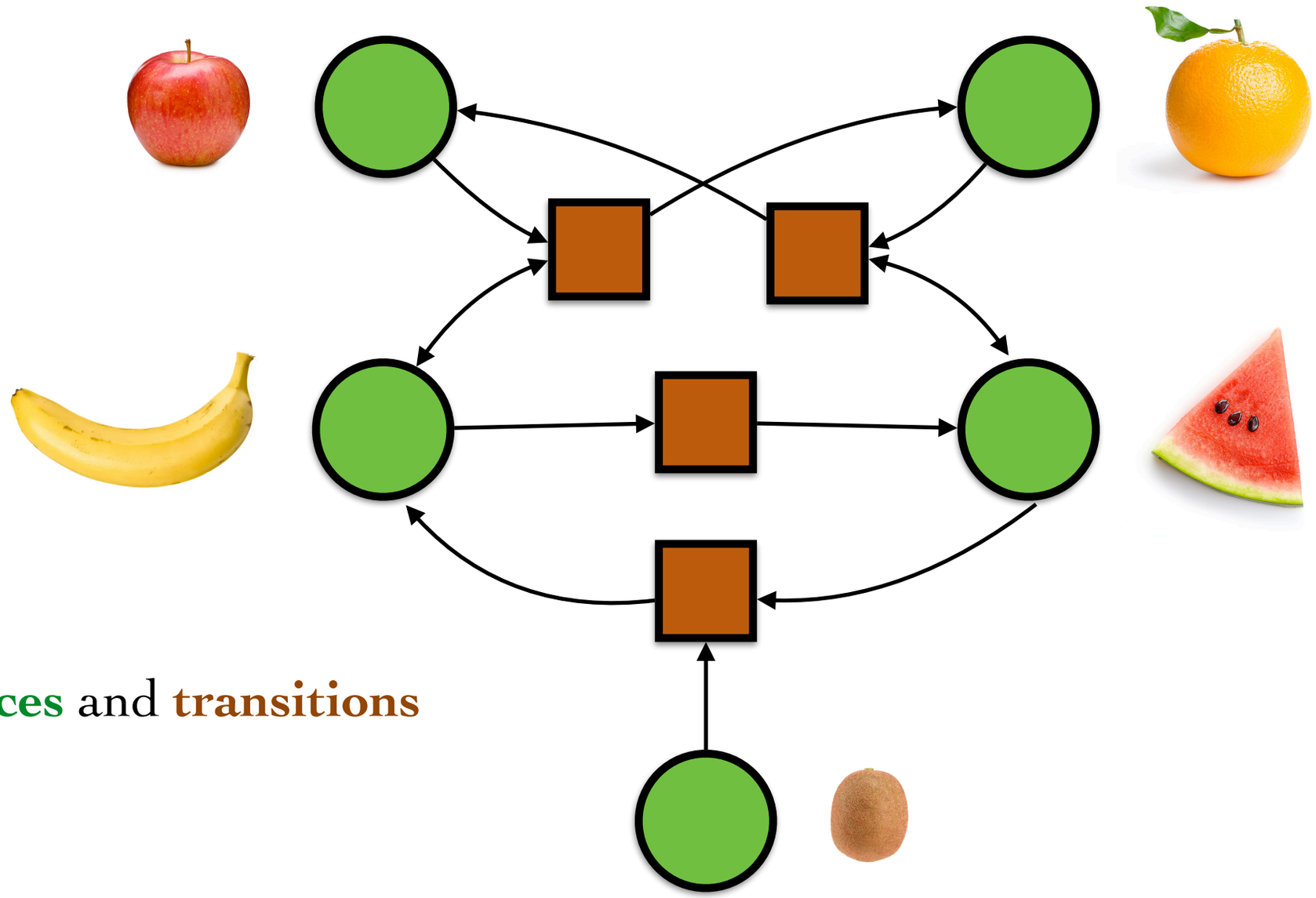
?

Petri nets



places and **transitions**

Petri nets



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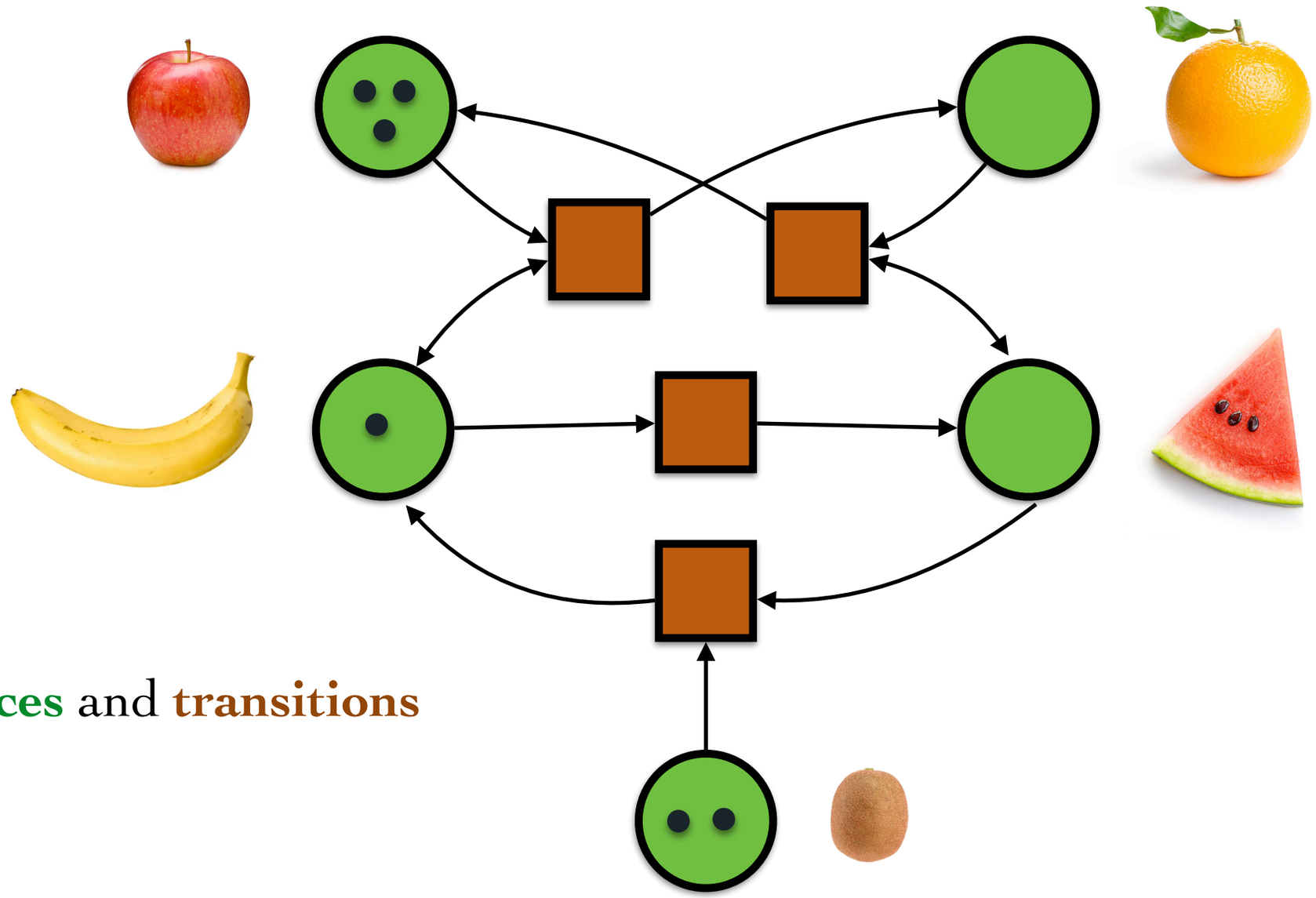

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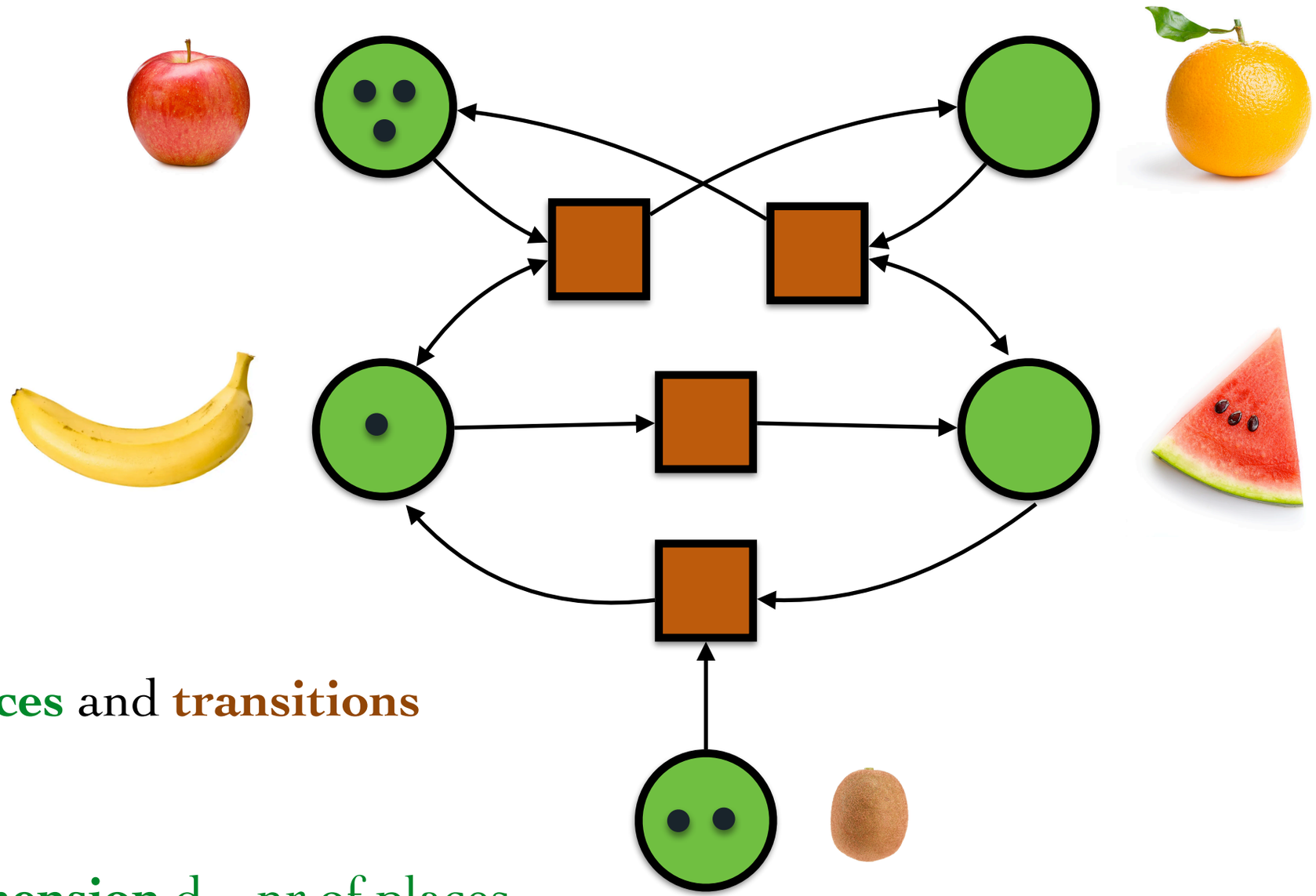
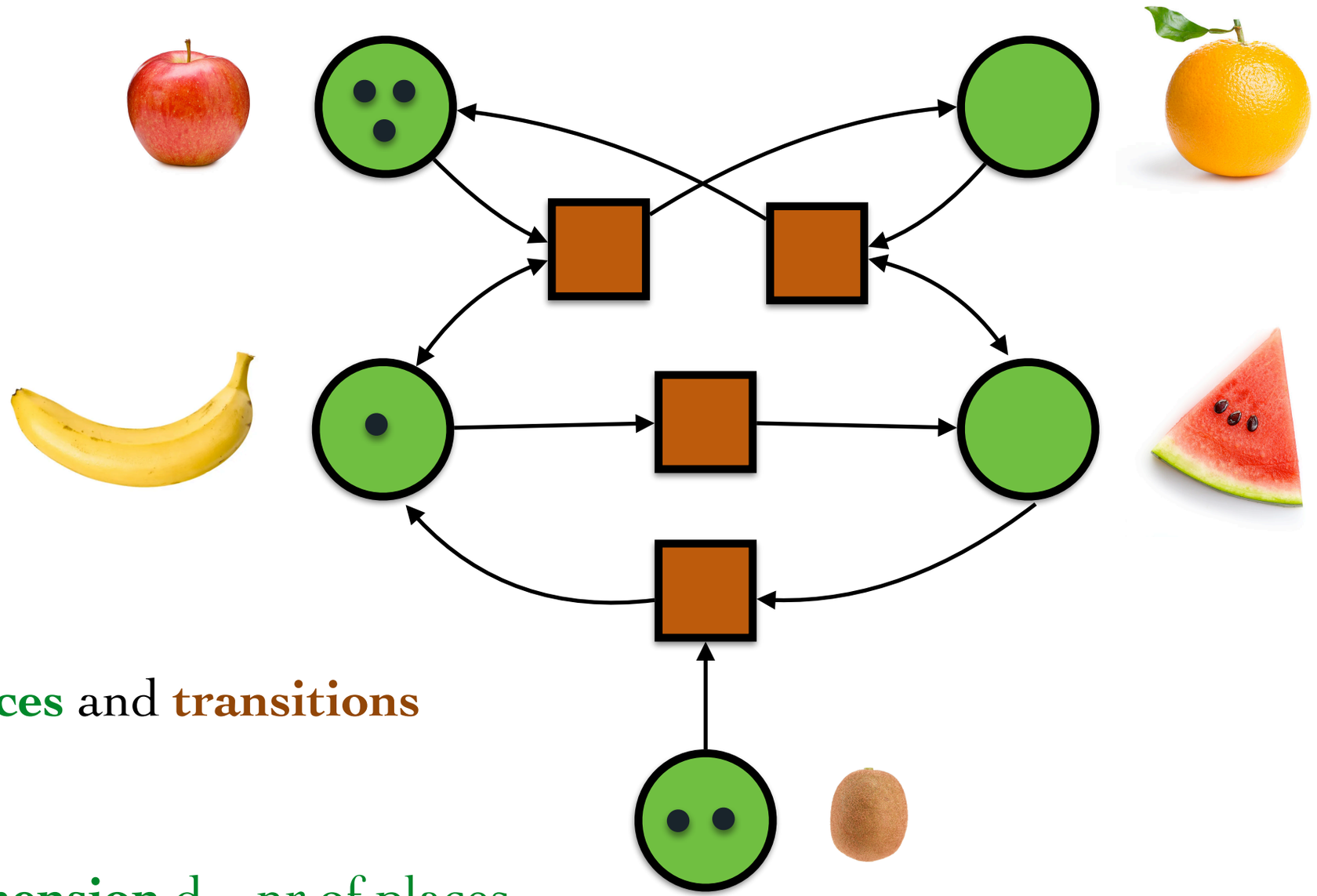


places and **transitions**

dimension d = nr of places

Petri nets

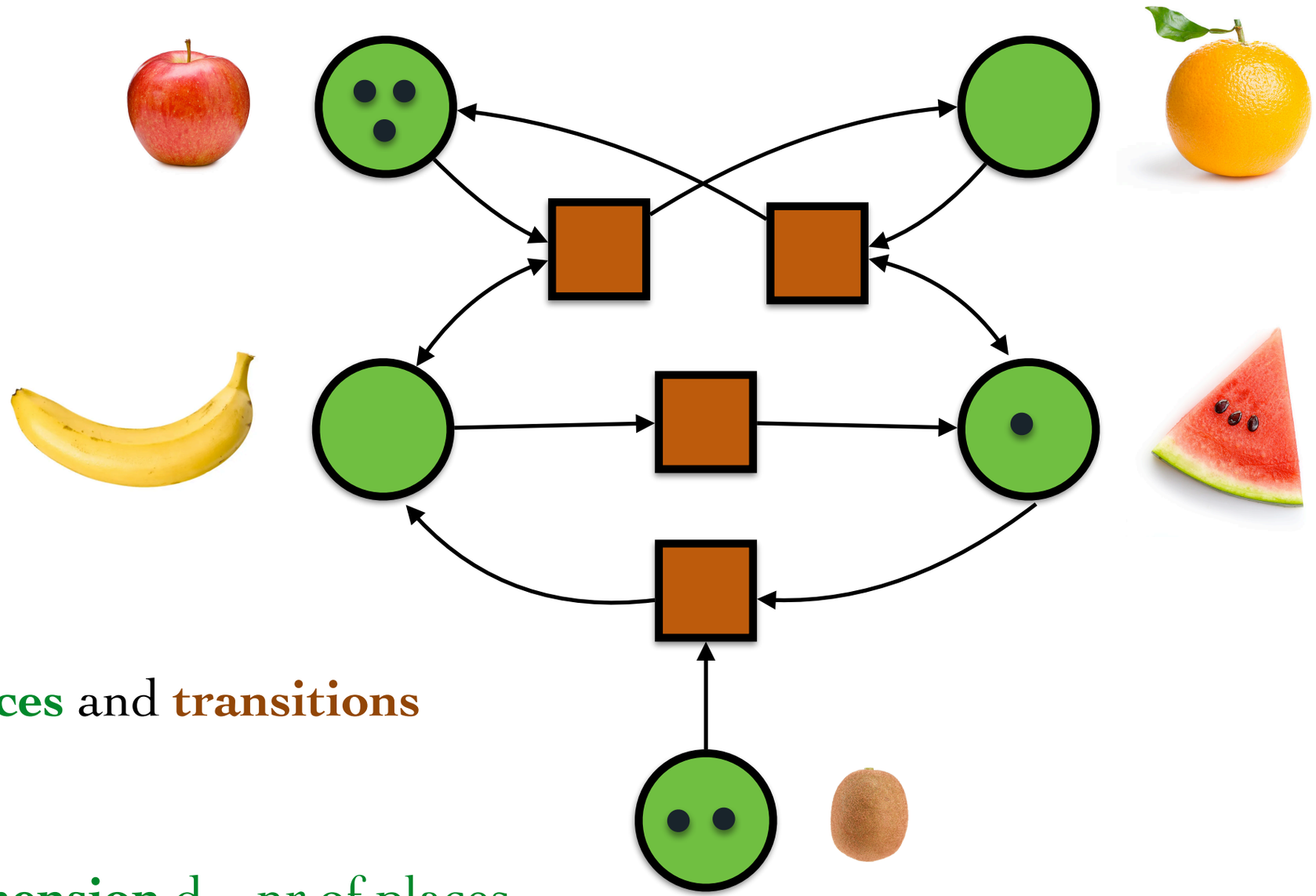


places and **transitions**

dimension d = nr of places

configuration : places $\rightarrow \mathbb{N} \equiv \mathbb{N}^d$

Petri nets



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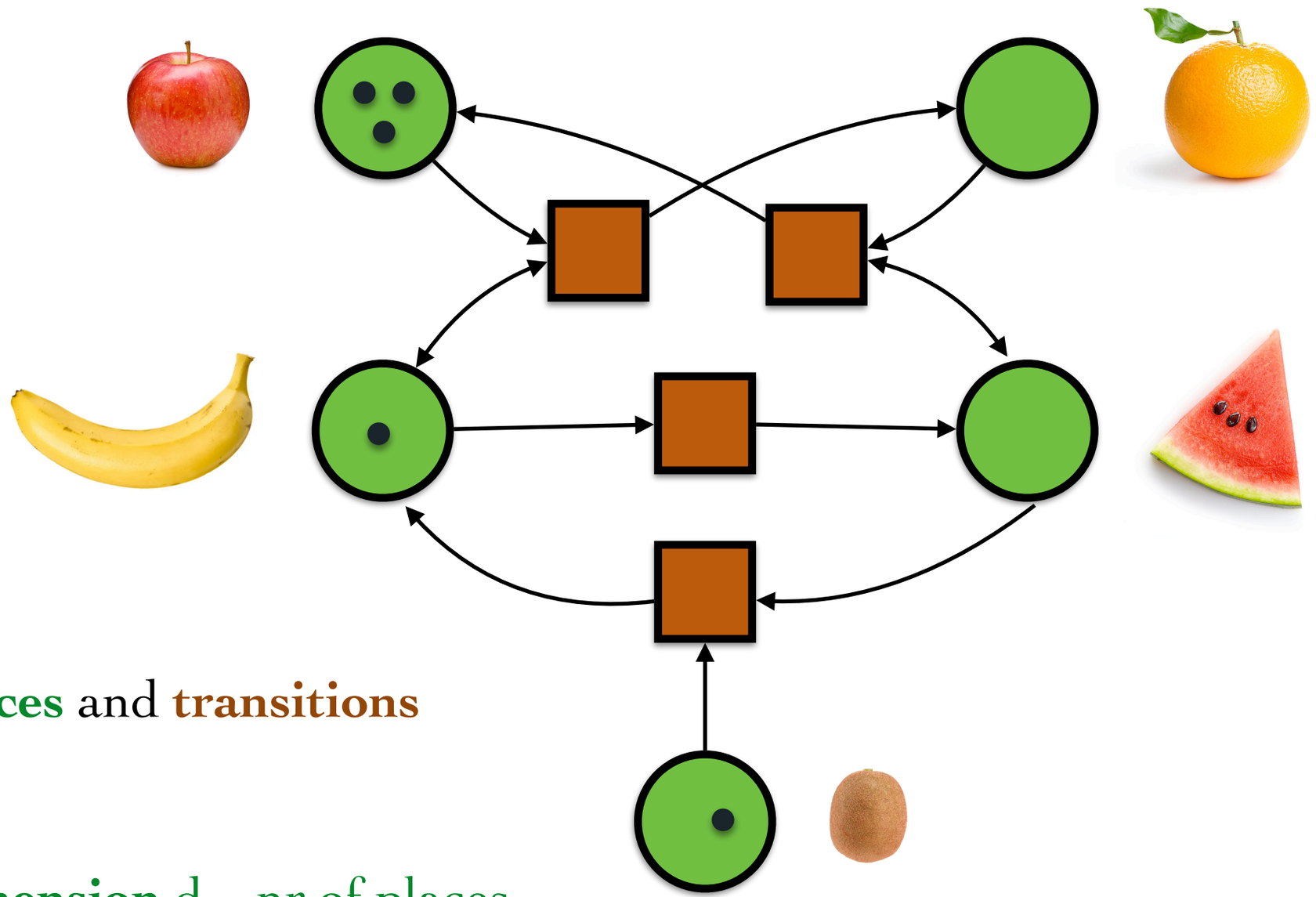
step relation between configurations

Petri nets



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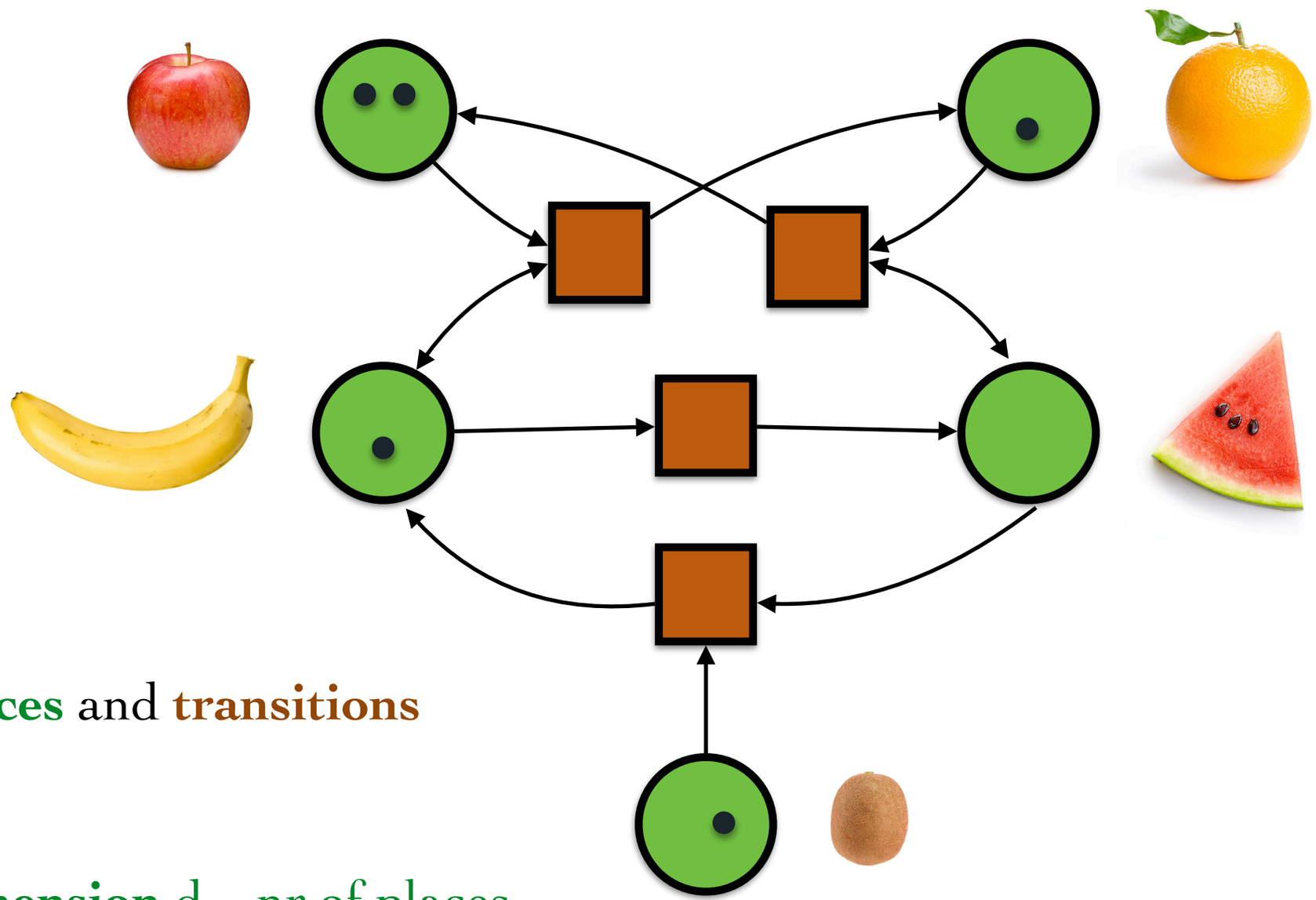

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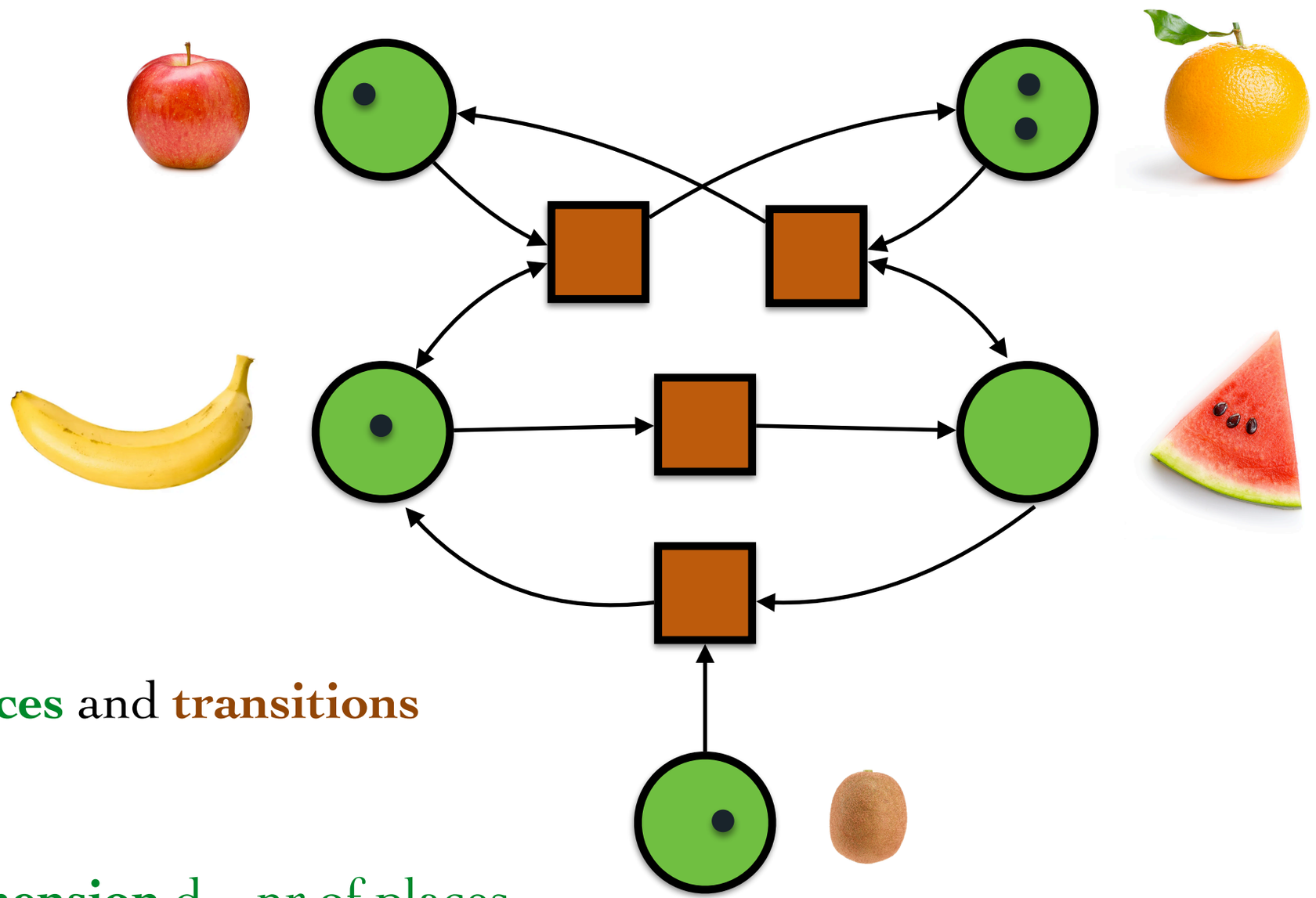

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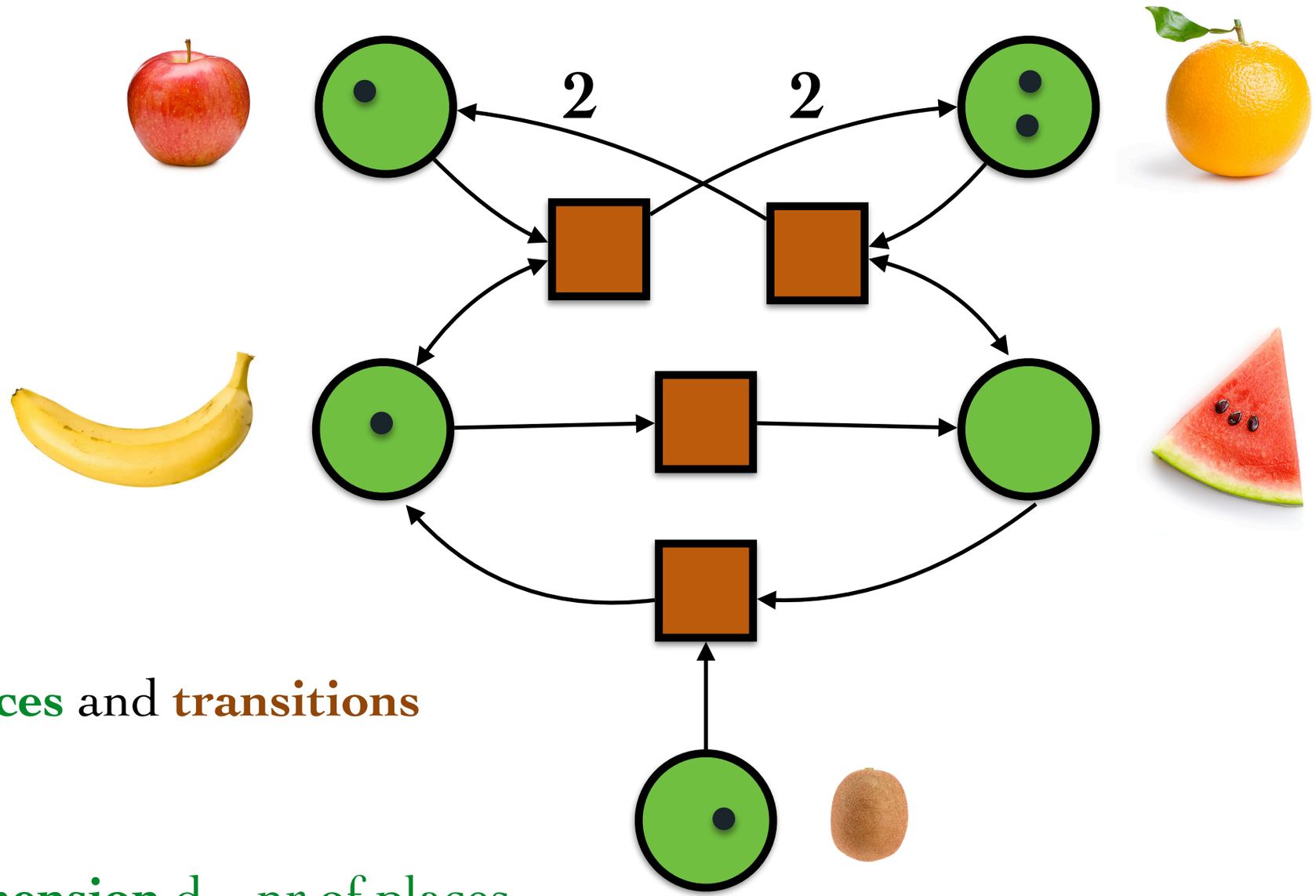

arcs weighted by $\mathbb{N}$

places and **transitions**

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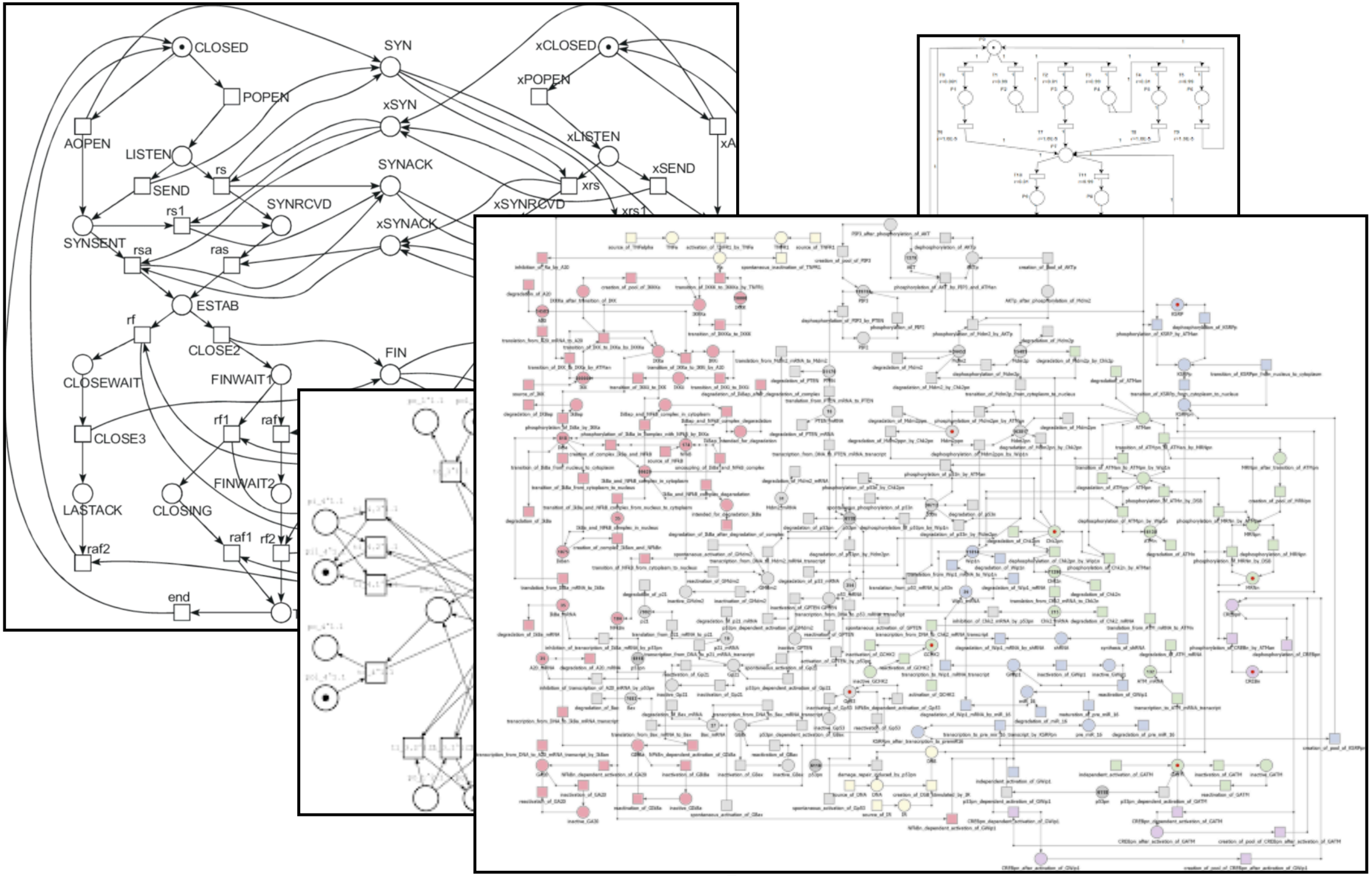
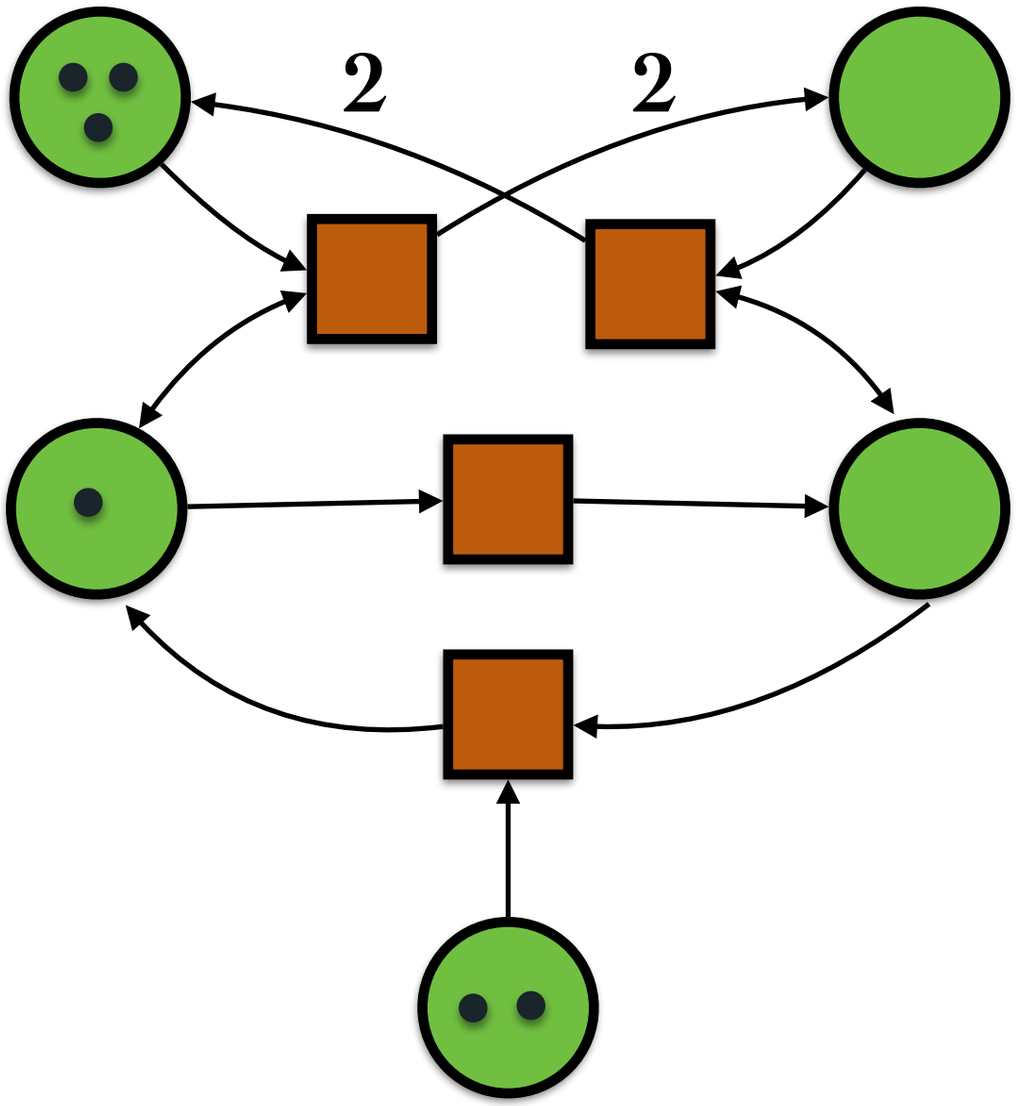
step relation between configurations

Useful model

Useful model



Reachability problem in Petri nets



configuration : $\mathbb{N}^d$

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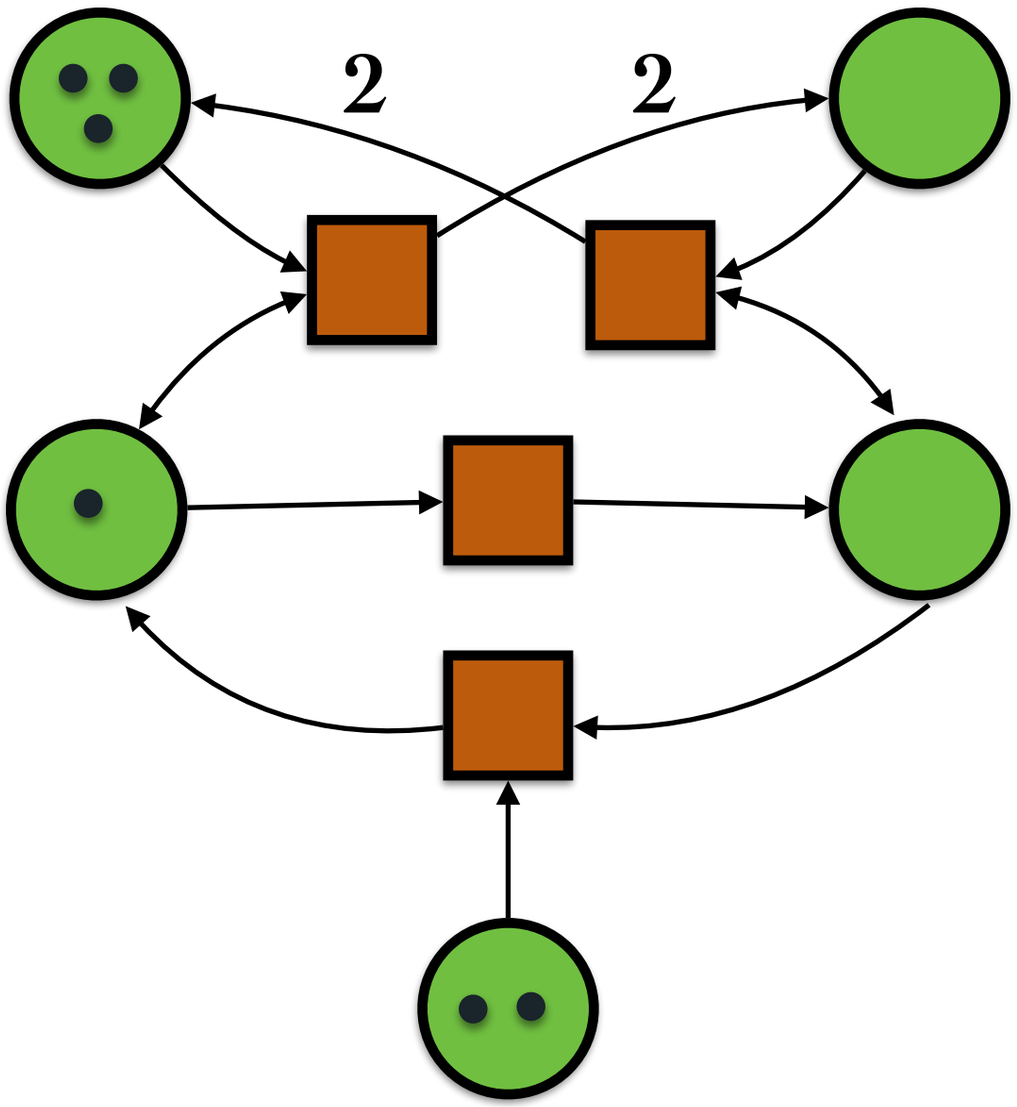
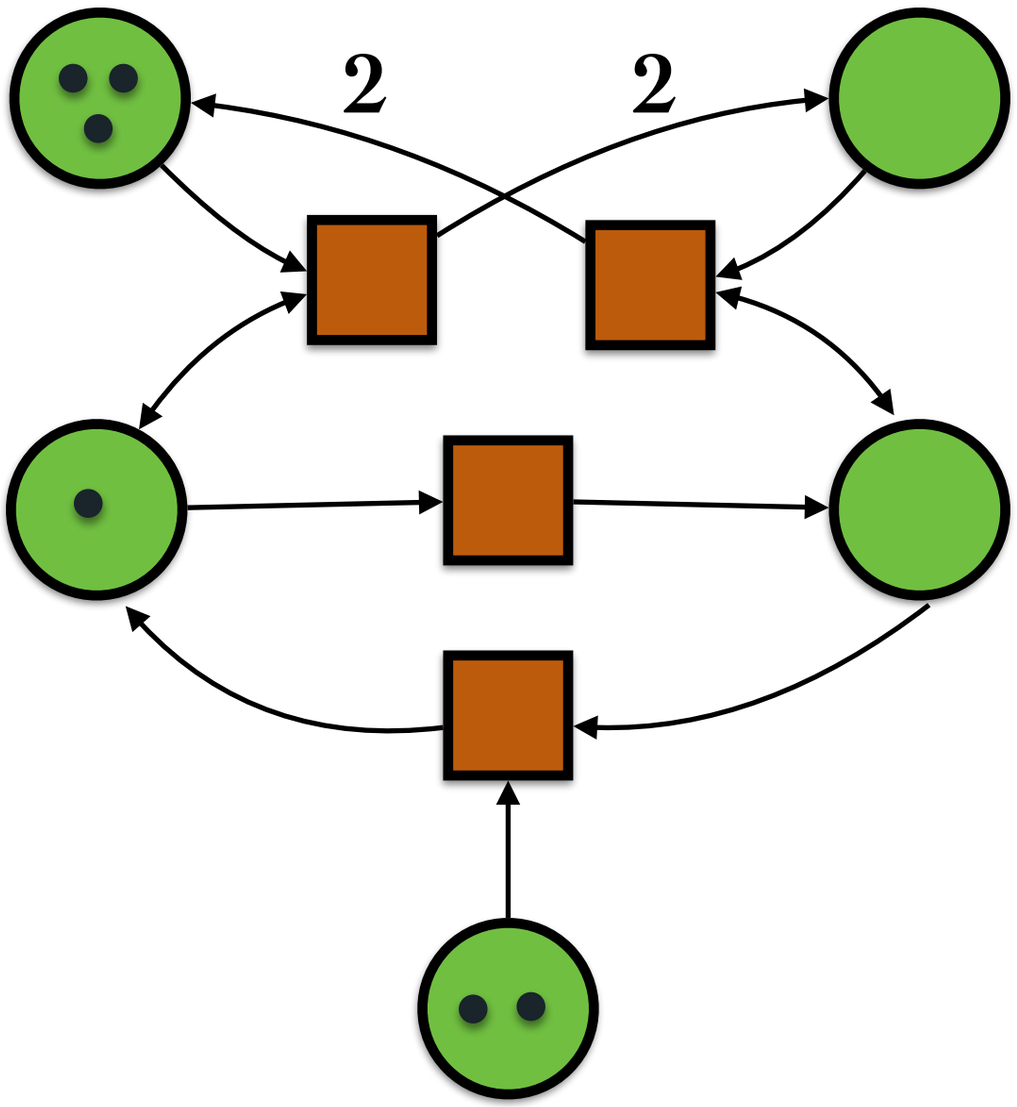


Decision problem:

configuration : $\mathbb{N}^d$

Reachability problem in Petri nets



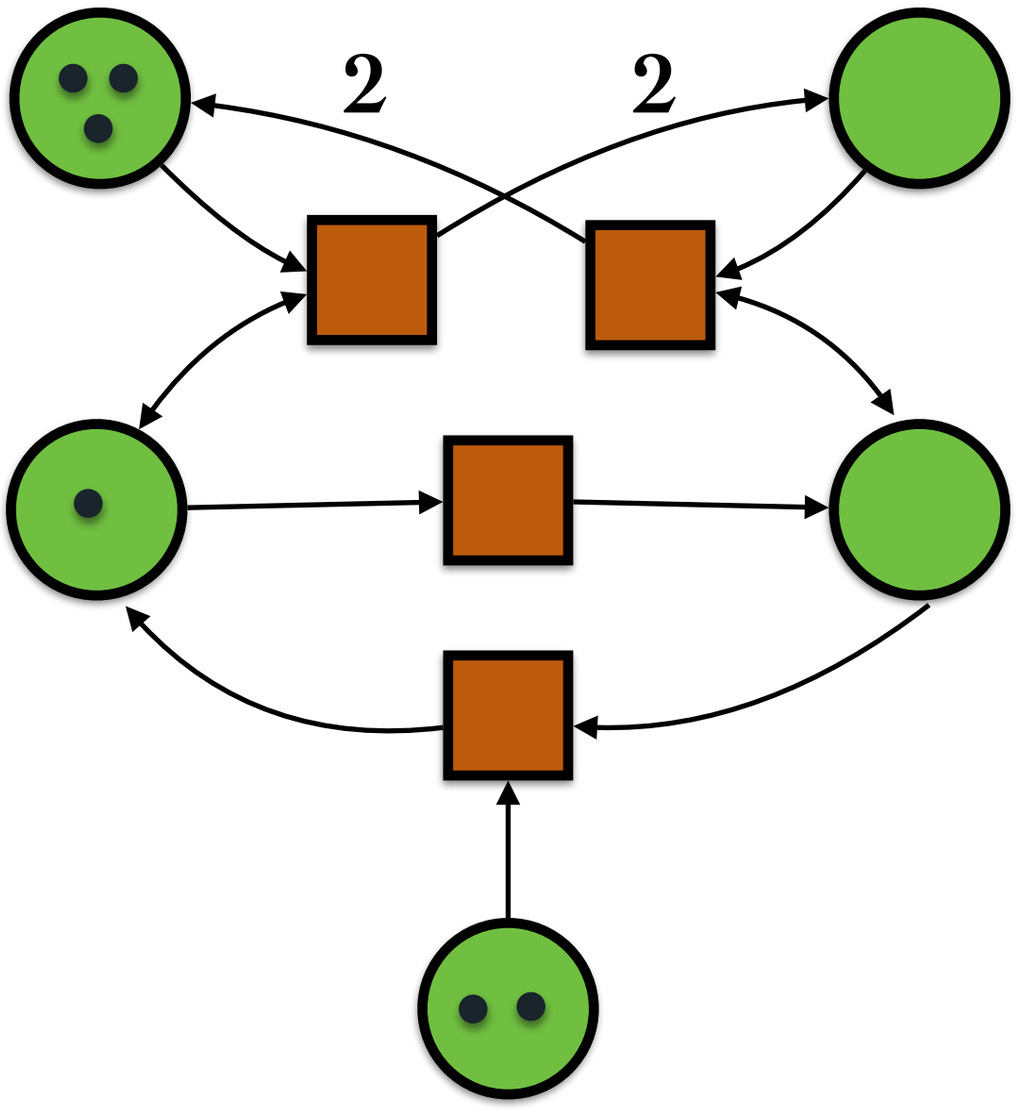
Decision problem:

given

- Petri net
- source configuration
- target configuration

configuration : $\mathbb{N}^d$

Reachability problem in Petri nets



configuration : $\mathbb{N}^d$

Decision problem:

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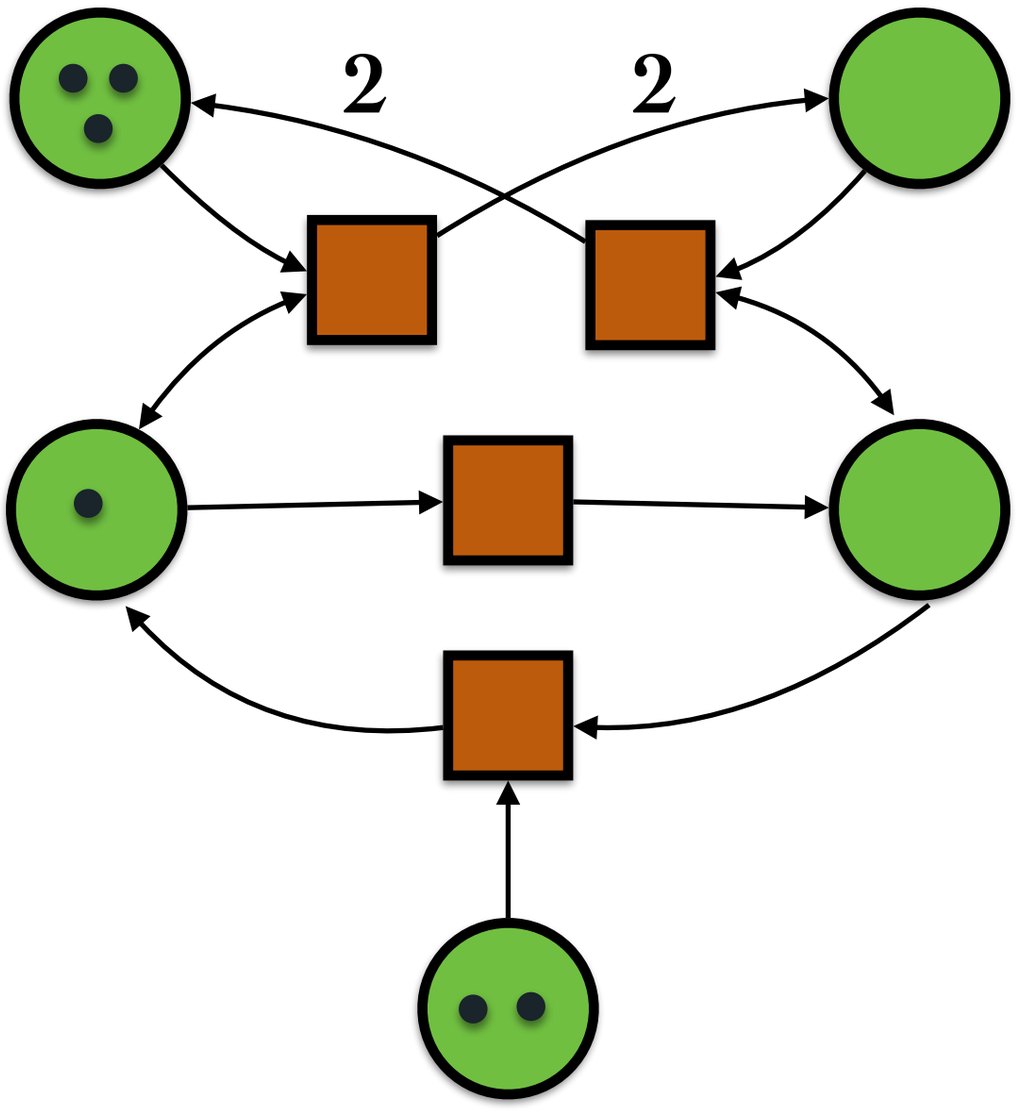
check if there is a sequence of steps (**run**) from source to target

~~Reachability~~ Coverability problem in Petri nets



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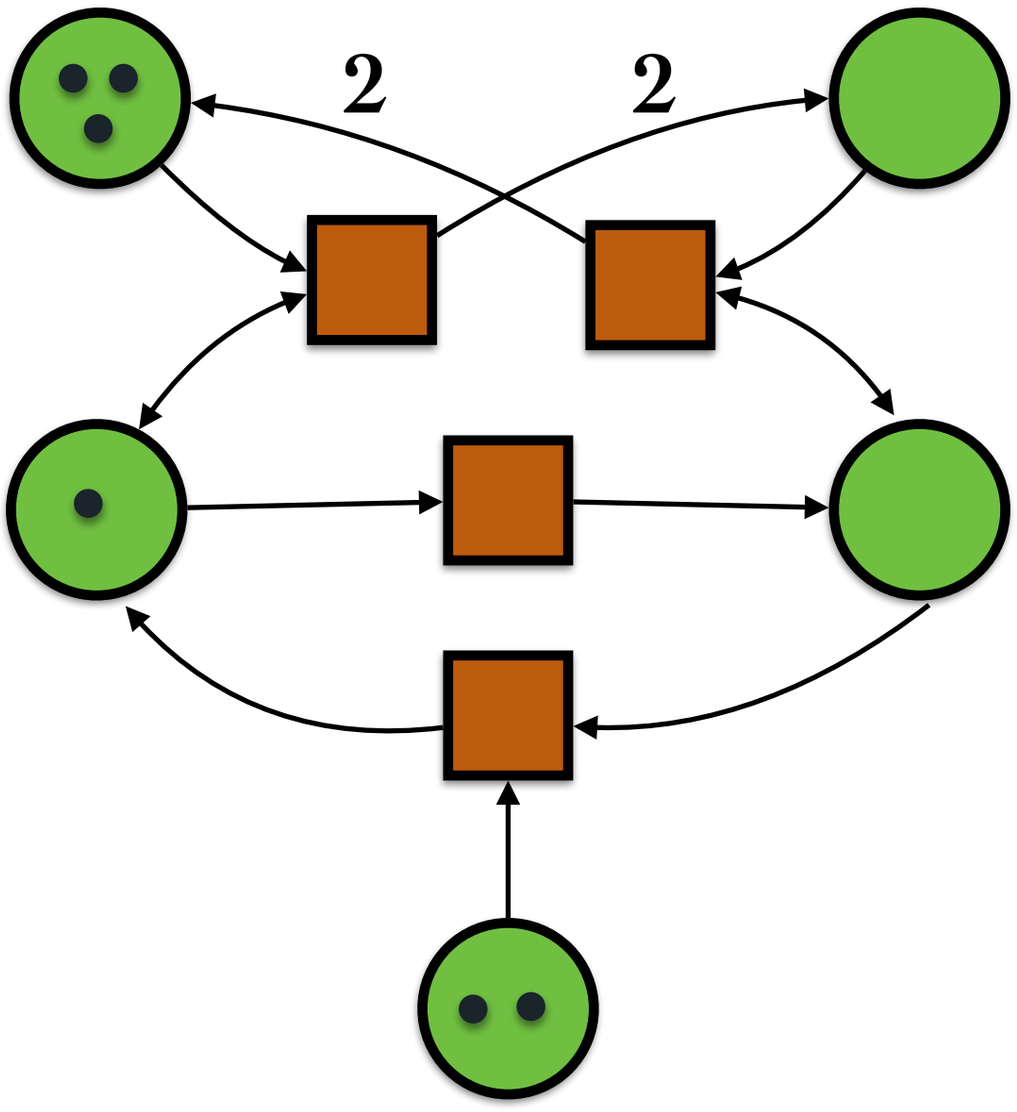

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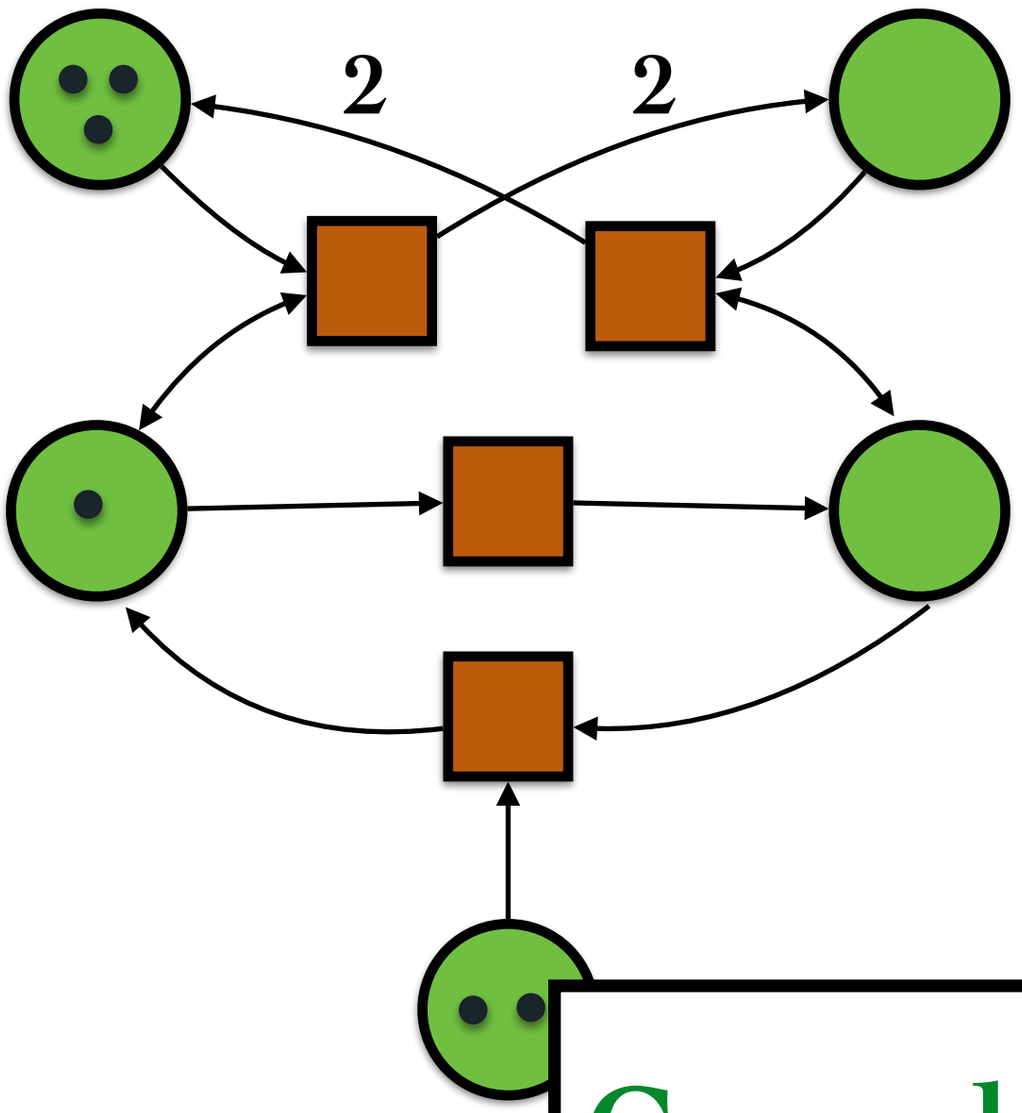

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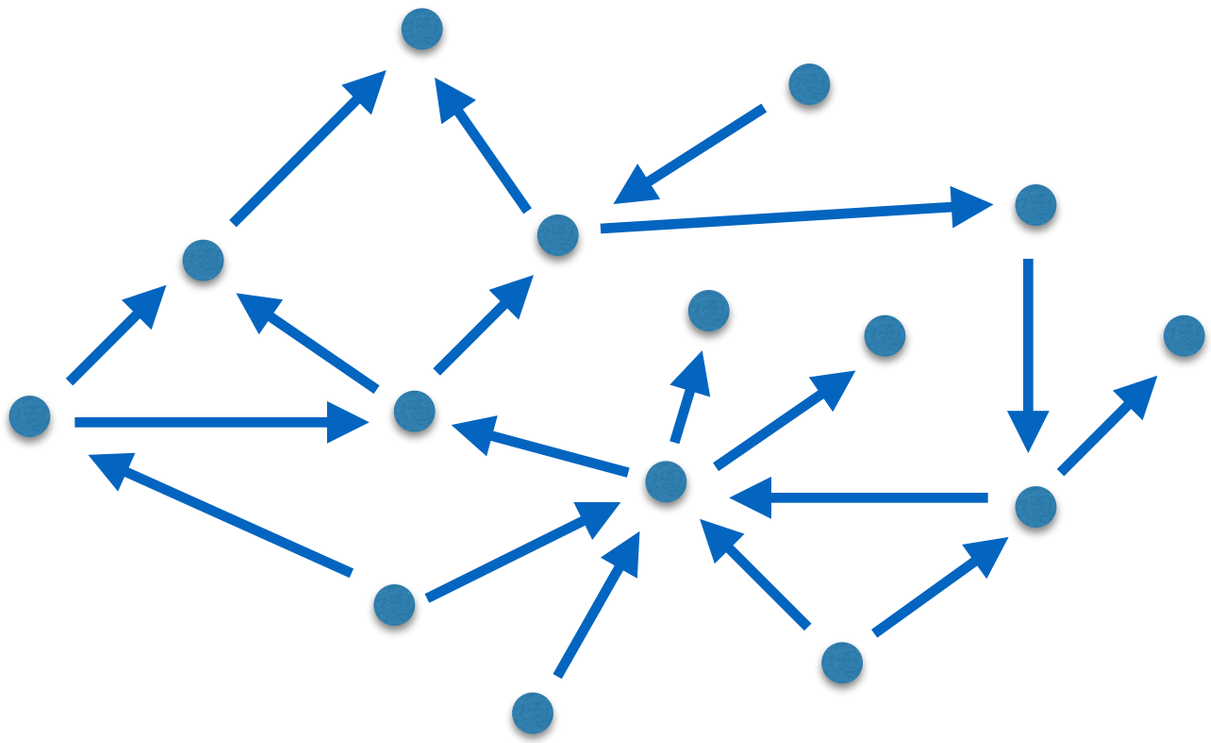
configuration : $\mathbb{N}^d$

Coverability $\rightsquigarrow$ **Reachability**

Reachability problem in Petri nets

configuration graph: configurations and steps



Reachability problem in Petri nets

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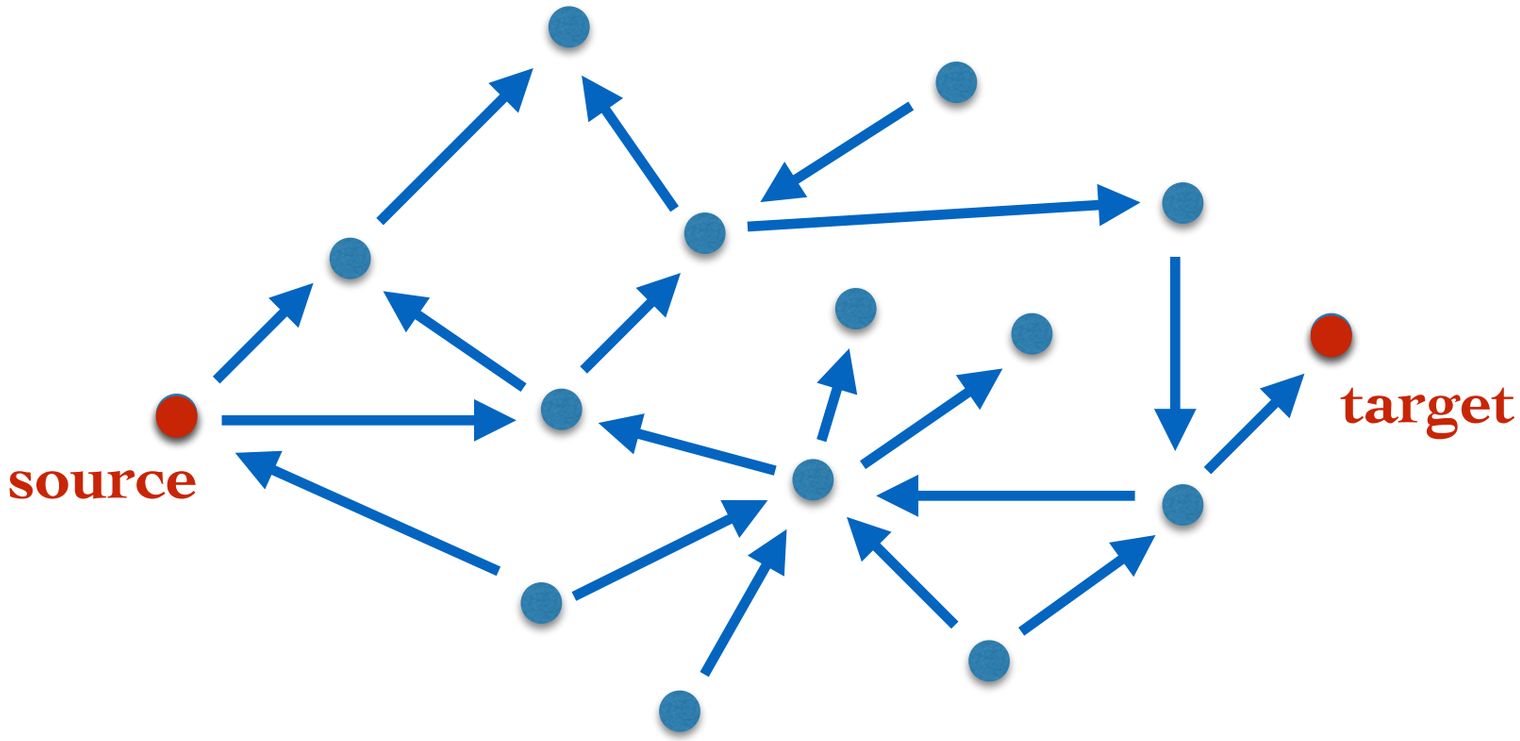

Reachability problem in Petri nets

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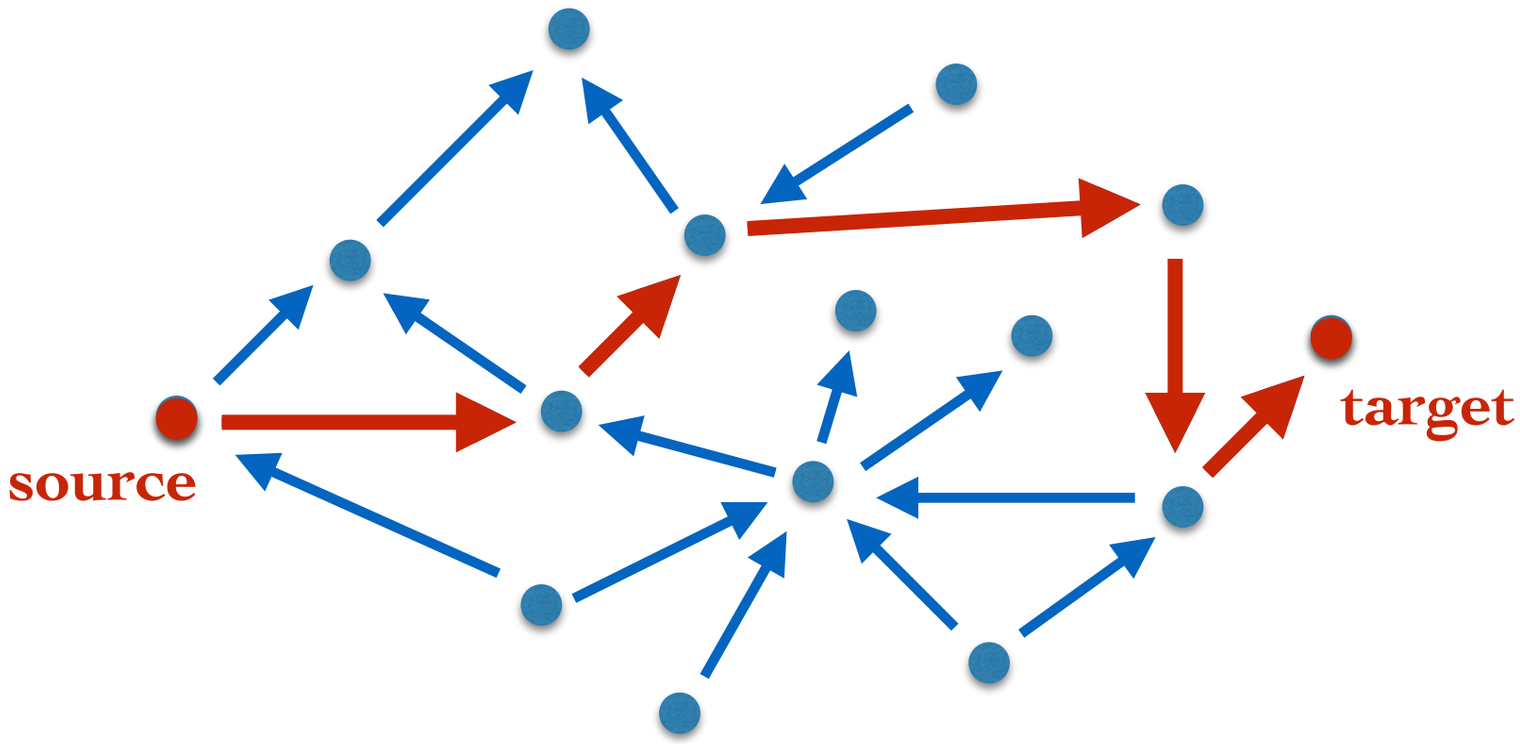
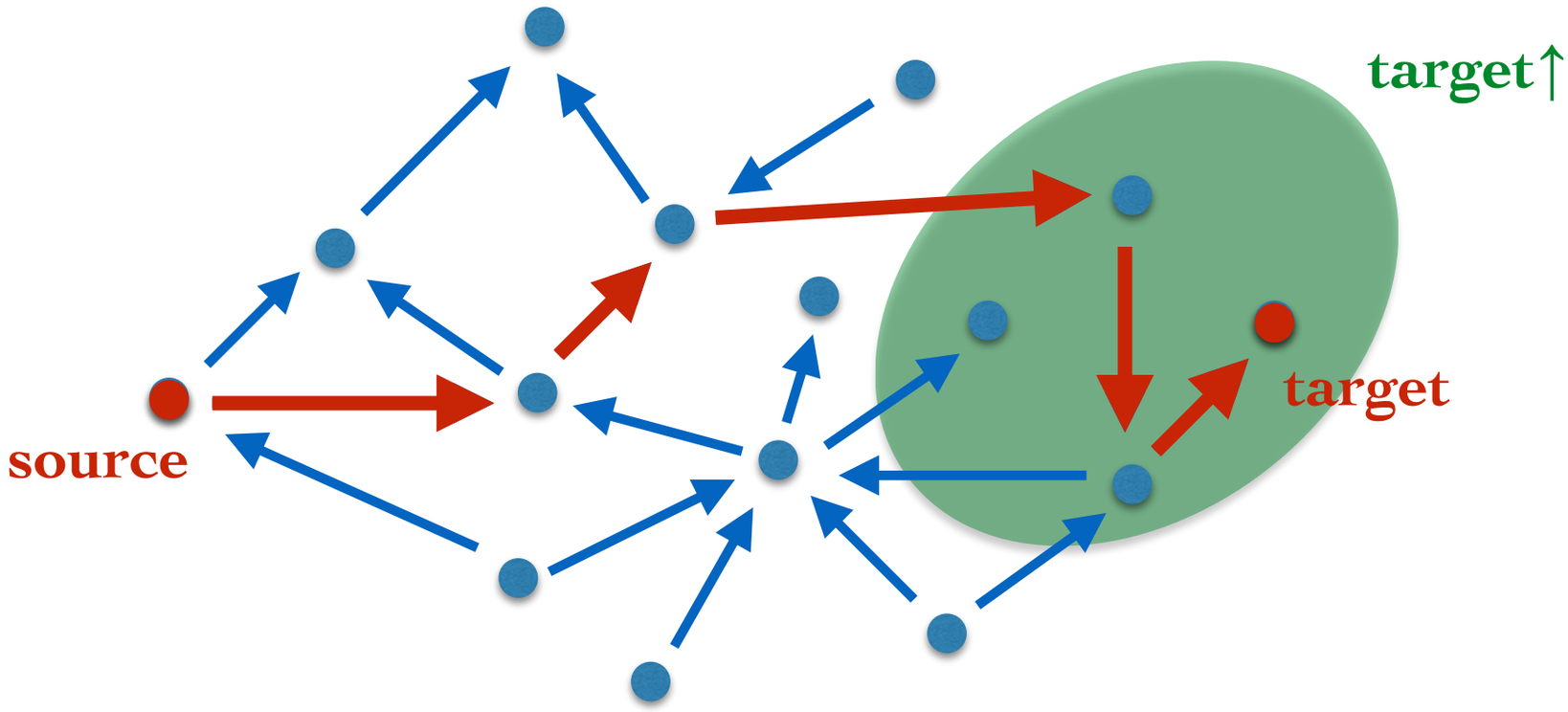


Reachability: is there a path (**run**) from source to target ?

~~Reachability~~ Coverability problem in Petri nets

configuration graph: configurations and steps



Reachability: is there a path (**run**) from source to target ?

Coverability: is there a path (**run**) from source to target↑ ?

Why is it important?

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- core verification problem

Why is it important?

- core verification problem
- equivalent to many other problems in
  - concurrency,
  - process algebra,
  - logic,
  - automata theory,
  - language theory,
  - linear algebra,
  - …

1969 decidability of coverability **[Karp, Miller]**

1970

1980

1990

2000

2010

2020

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1976 — EXPSPACE lower bound for coverability **[Lipton]**

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1992 — decidability of reachability - refined data structure **[Lambert]**

KLM decomposition

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KLM decomposition

huge complexity gap!

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KLM decomposition

huge complexity gap!

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2011 — decidability of reachability by semilinear invariants **[Leroux]**

2015 — upper bound $\mathbf{F}_{\omega^3}$ **[Leroux, Schmitz]**

2019 — Ackermannian upper bound $\mathbf{F}_{\omega}$ **[Leroux, Schmitz]**

2020

2019 — Ackermann upper bound $\mathbf{F}_\omega$ **[Leroux, Schmitz]**

2020

2019 — Ackermann upper bound F_ω **[Leroux, Schmitz]**

2019 — TOWER lower bound F_3 **[Czerwiński, L., Lazic, Leroux, Mazowiecki]**

2020

$$2^{2^{2^{\cdots^{2}}}\Big\}n}$$

2019 — Ackermann upper bound $\mathbf{F}_\omega$ **[Leroux, Schmitz]**

2019 — TOWER lower bound $\mathbf{F}_3$ **[Czerwiński, L., Lazic, Leroux, Mazowiecki]**

2020

$$2^{2^{2^{\cdots^{2}}}} \Big\} n$$

2021 — super-TOWER lower bound **[Czerwiński, L., Orlikowski]**

$$2^{2^{2^{\cdots^{2}}}} \Big\} 2^n$$

2019 — Ackermann upper bound F_ω **[Leroux, Schmitz]**

2019 — TOWER lower bound F_3 **[Czerwiński, L., Lazic, Leroux, Mazowiecki]**

2020

$$\left.2^{2^{2^{\cdots^{2}}}}\right\} n$$

2021 — super-TOWER lower bound **[Czerwiński, L., Orlikowski]**

$$\left.2^{2^{2^{\cdots^{2}}}}\right\} 2^n$$

gap closed!

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$$2^{2^{2^{\cdots^2}}} \Big\} n$$

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Fast growing functions and the induced complexity classes

gap closed!

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Fast growing functions and the induced complexity classes

$$A_1(n) = 2n$$

$$A_{i+1}(n) = \underbrace{A_i \circ A_i \circ \ldots \circ A_i}_{n}(1) = A_i^{n}(1)$$

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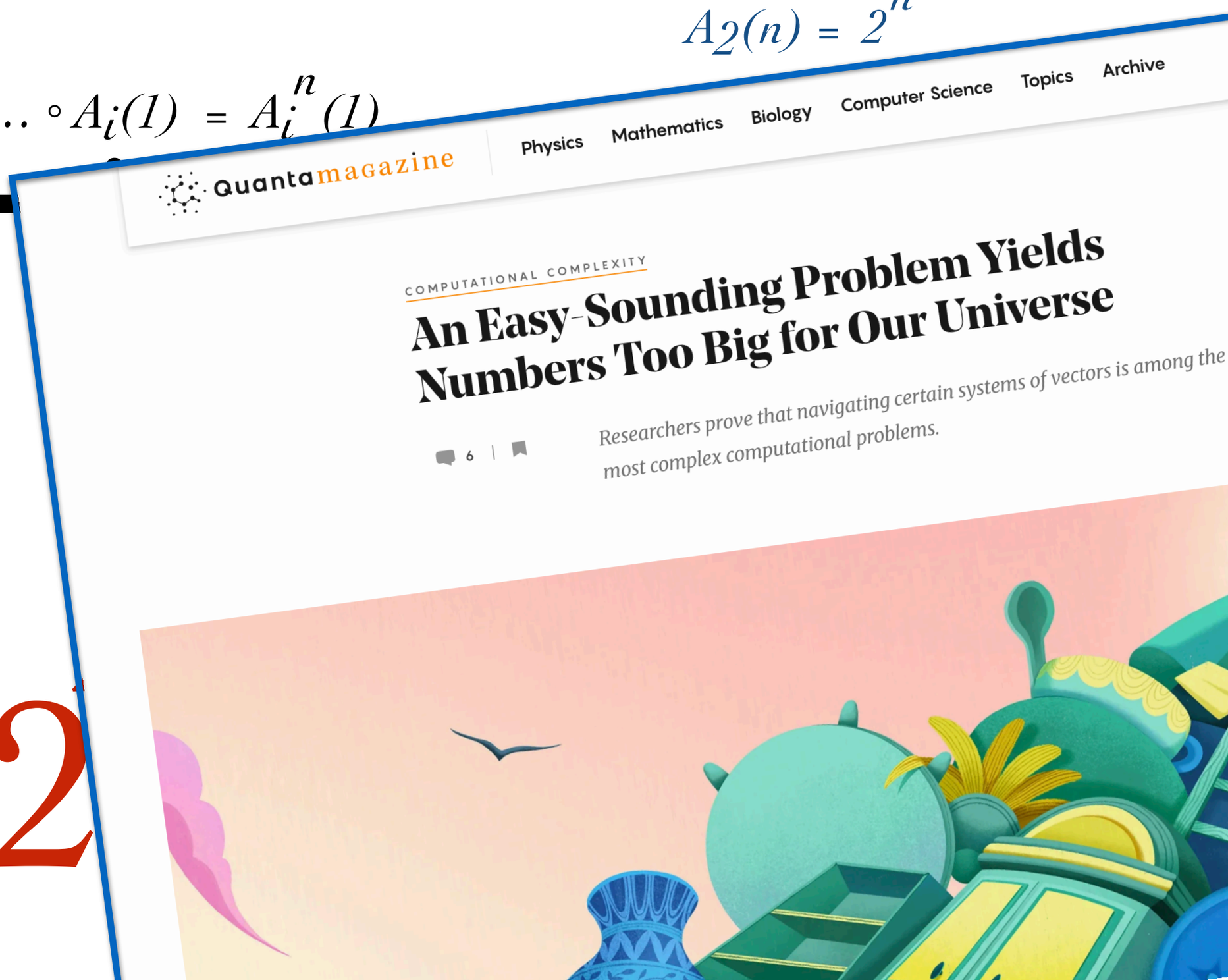

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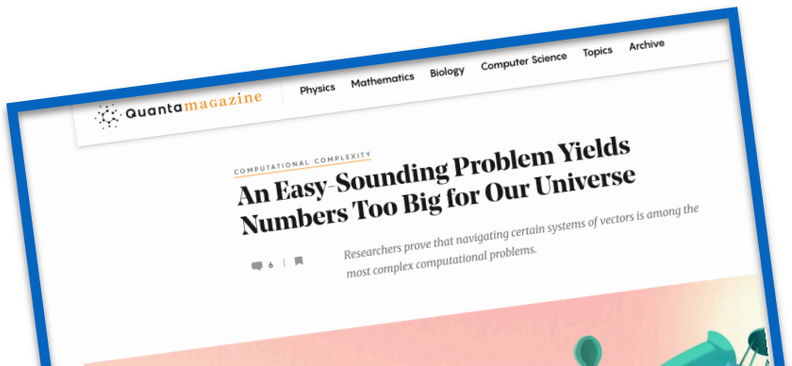

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$$A_\omega(n) = A_n(n)$$ Ackermann function

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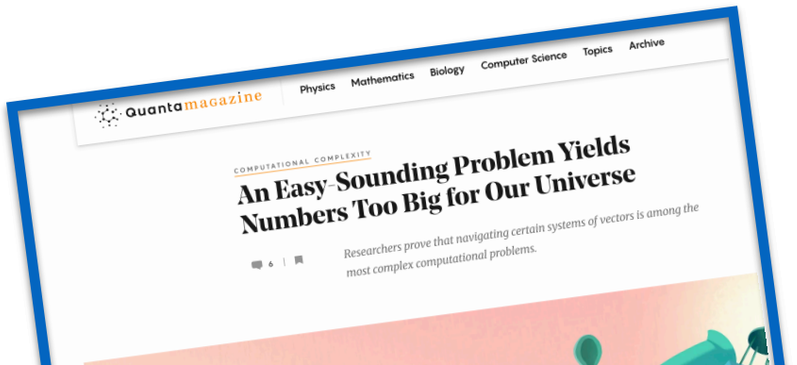

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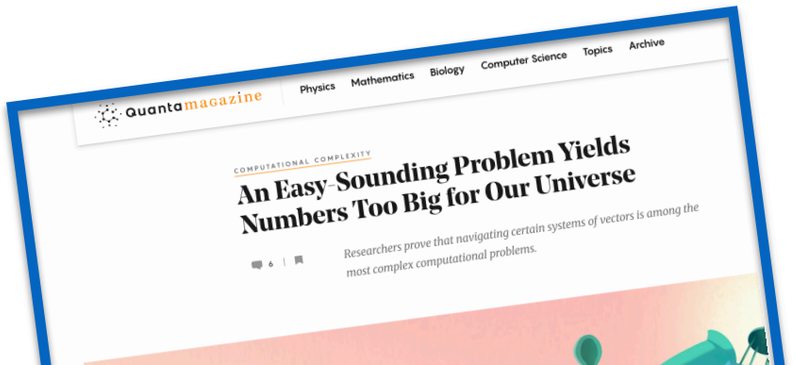
$$A_\omega(n) = A_n(n) \quad \text{Ackermann function}$$

$$F_i = \bigcup_{j_1 \ldots j_m < i} \text{DTIME}(A_i \circ A_{j_1} \circ \ldots \circ A_{j_m})$$



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$$\ldots$$

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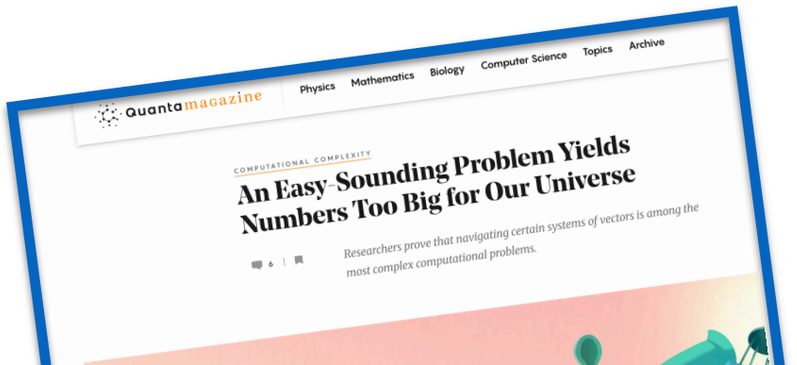

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$$A_2(n) = 2^n$$

$A_3(n) =$ tower(n)

$$= 2^{2^{\cdot^{\cdot^{\cdot^{2}}}}} \Big\}\, n$$

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…

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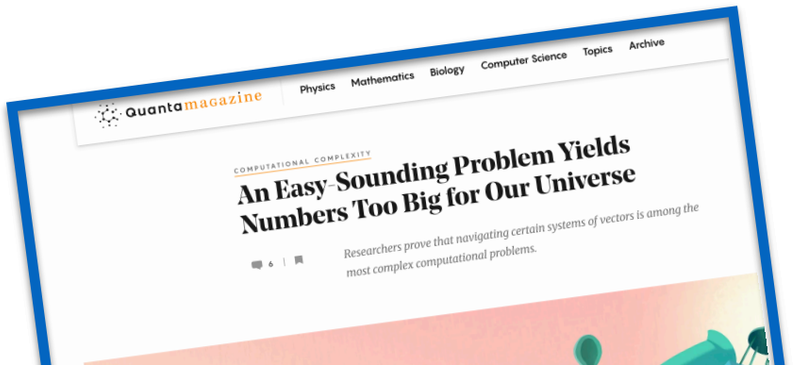
Hierarchy goes beyond ω !



Other decision problems

- boundedness
- place boundedness
- termination
- (in)evitability
- repeated coverability

- deadlock-freedom
- liveness
- semilinearity
- regular separability

- regularity / context freeness
- model-checking
- equivalence checking

Other decision problems

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- place boundedness
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EXPSPACE-complete

- deadlock-freedom
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Ackermann-hard

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undecidable

Other decision problems

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reachability

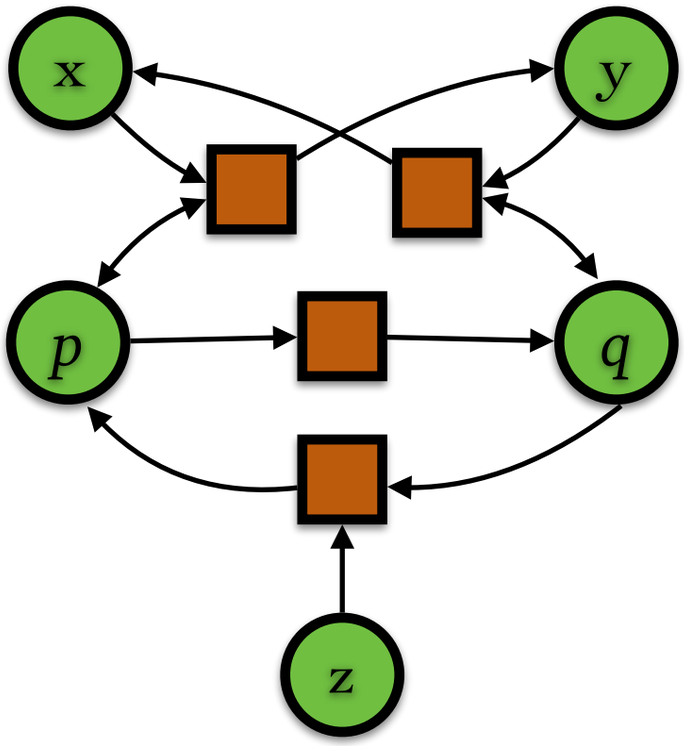
and

coverability

only

Many faces of Petri nets

- Petri nets:



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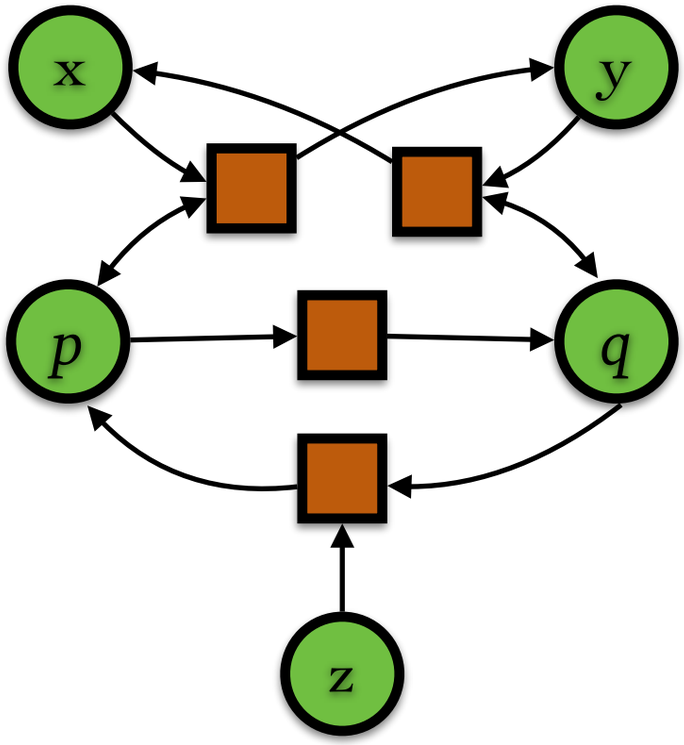
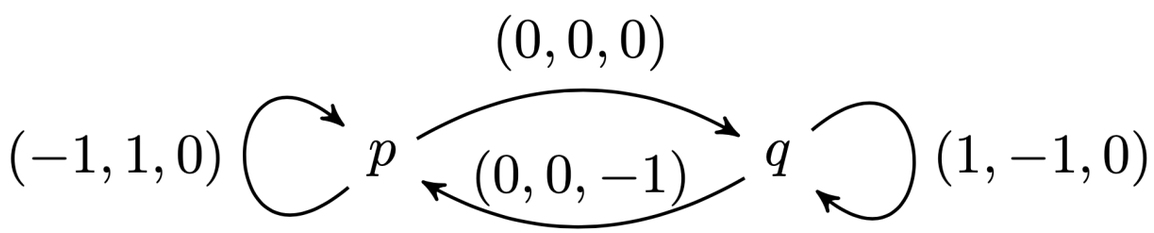


- vector addition systems with states (VASS):



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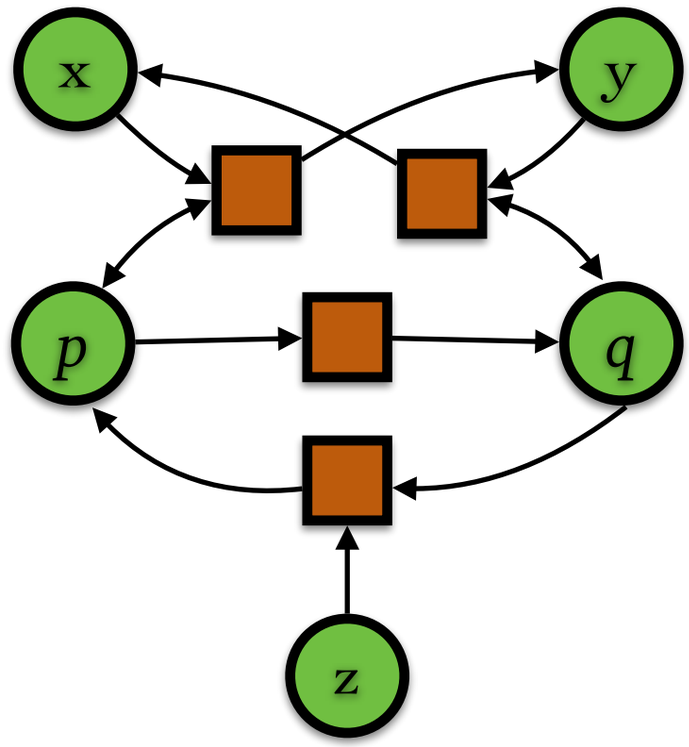

- counter programs **without zero-tests**:

```
1: loop
2:     loop
3:         x −= 1    y += 1
4:     loop
5:         x += 1    y −= 1
6:     z −= 1
```

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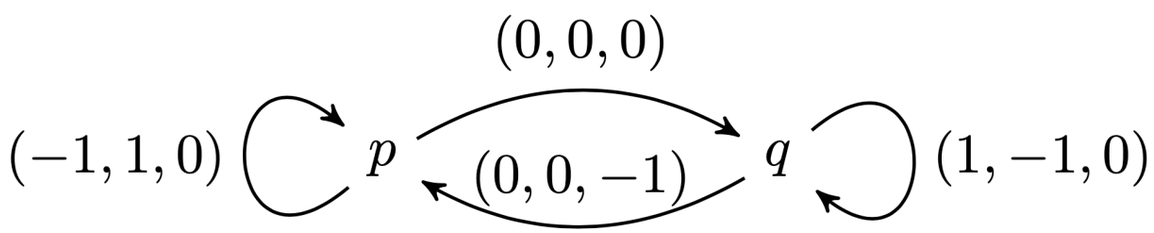

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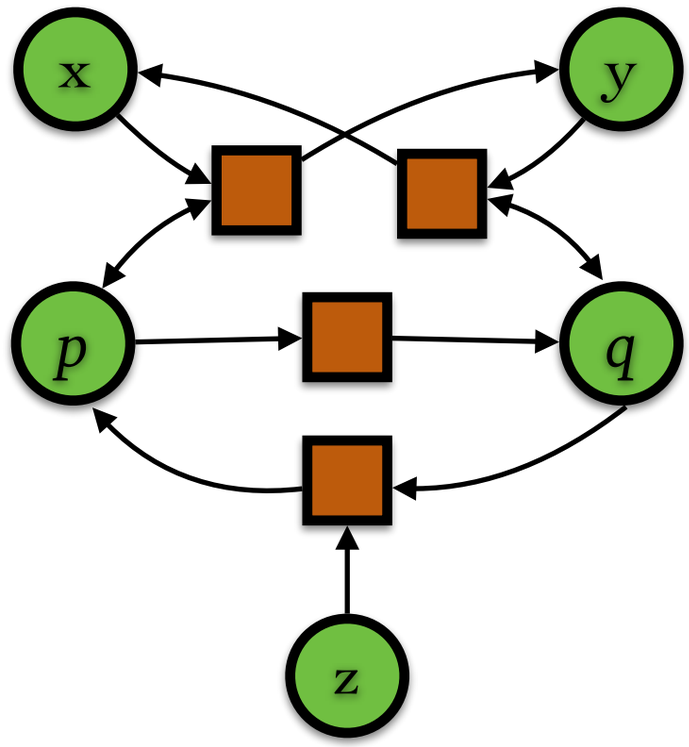

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```
1:  goto 2 or 10
2:  goto 3 or 5
3:  x −= 1    y += 1
4:  goto 2
5:  goto 6 or 8
6:  x += 1    y −= 1
7:  goto 5
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9:  goto 1
10: ...
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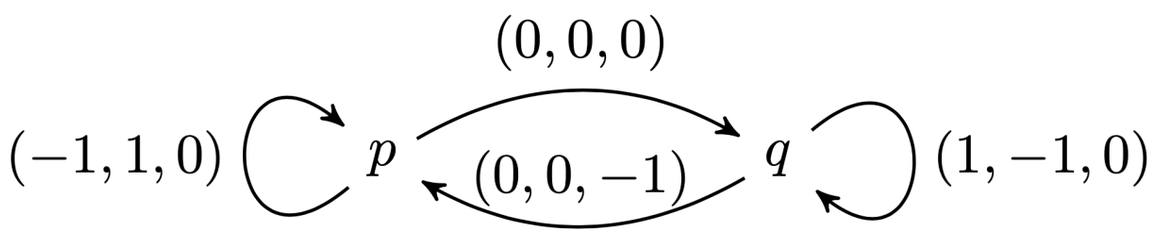

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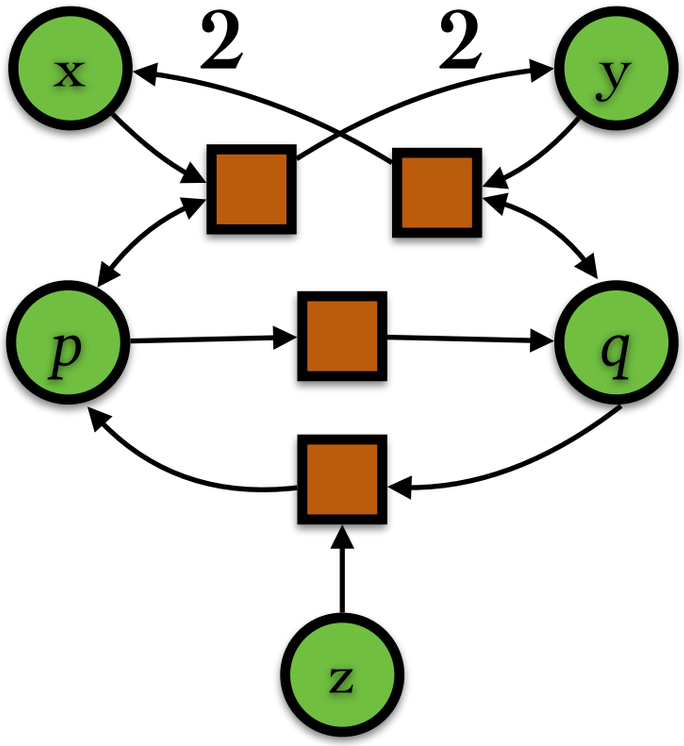

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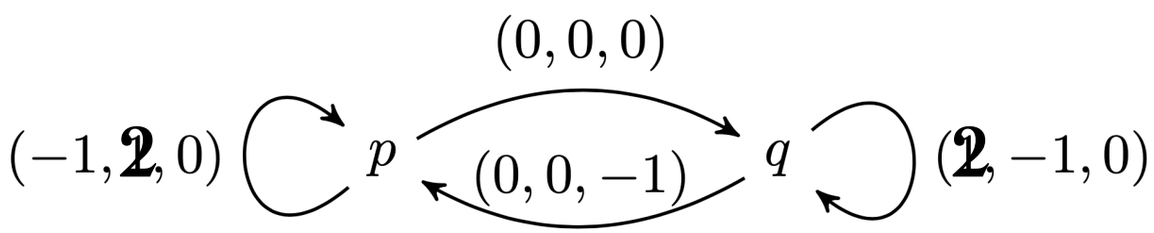

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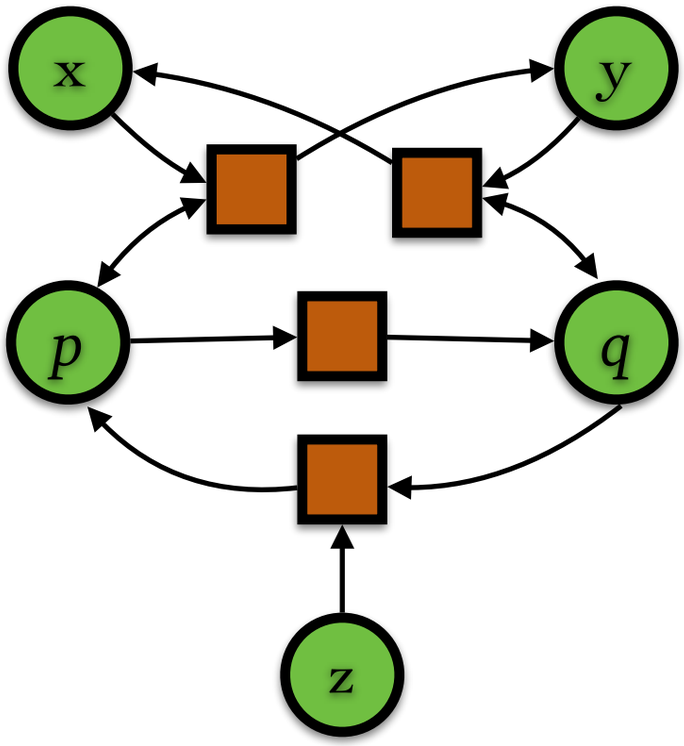

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counter programs with zero-tests are Turing-complete

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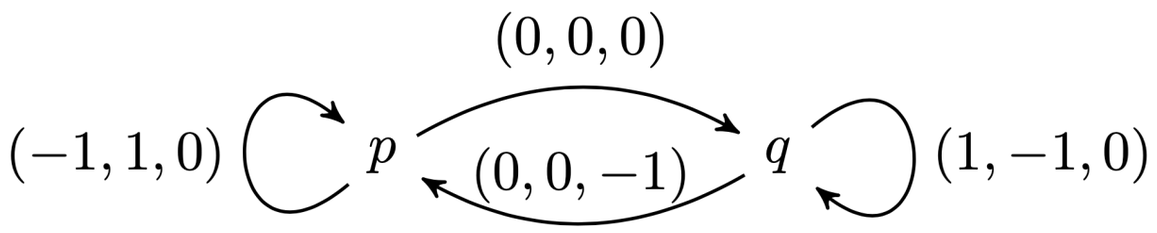

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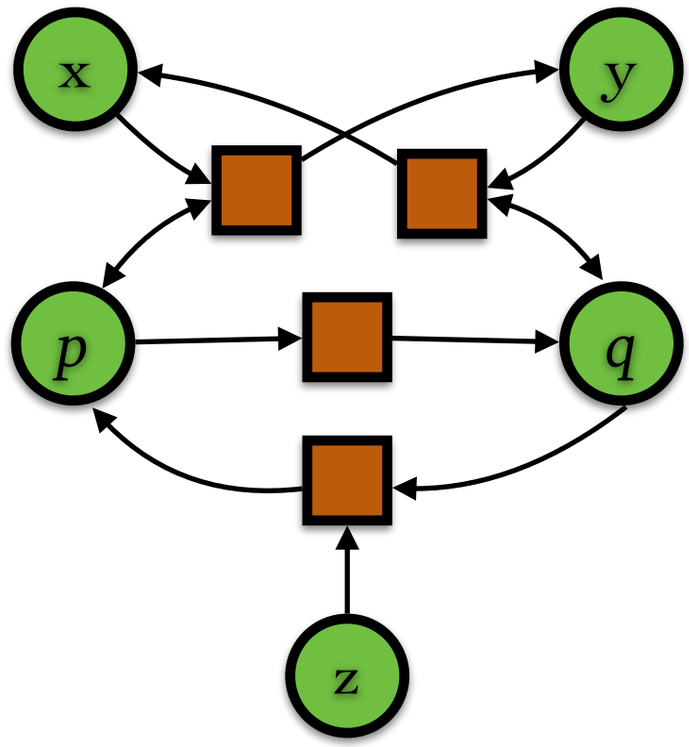

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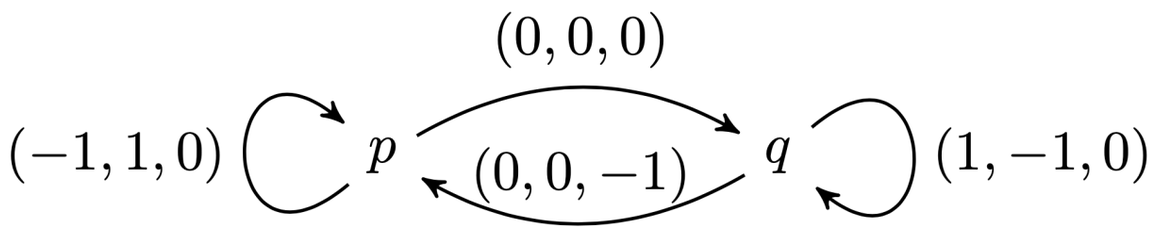

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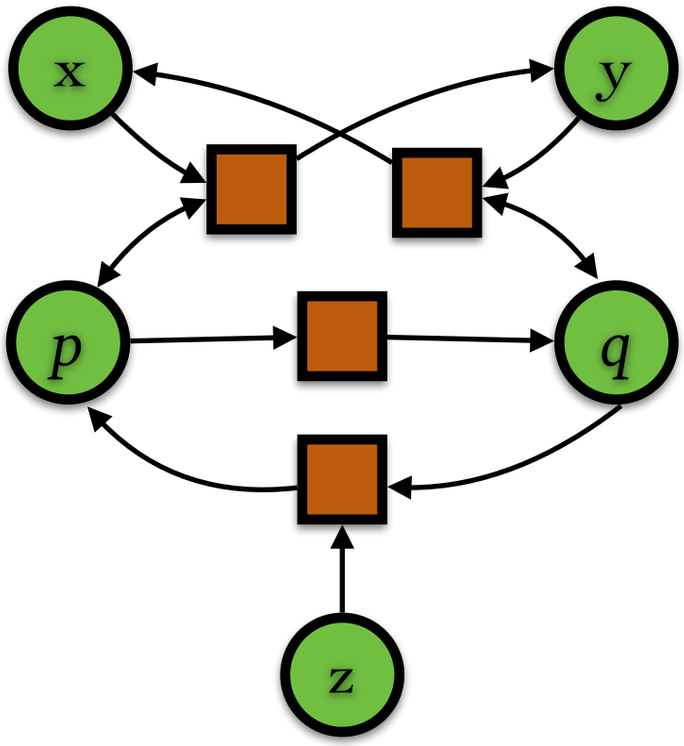

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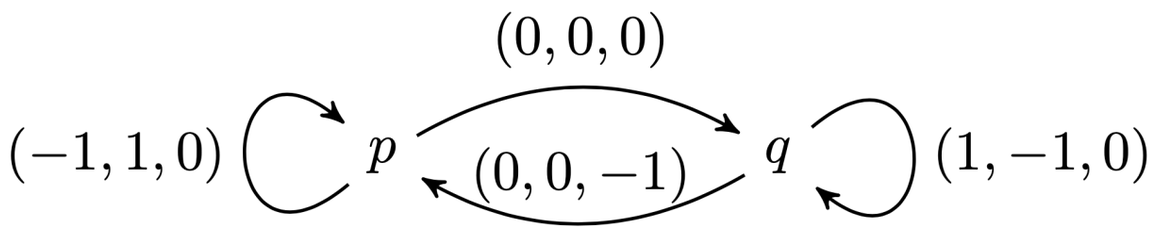

- vector addition systems
- counter automata **without zero-tests**
- ground multiset rewriting
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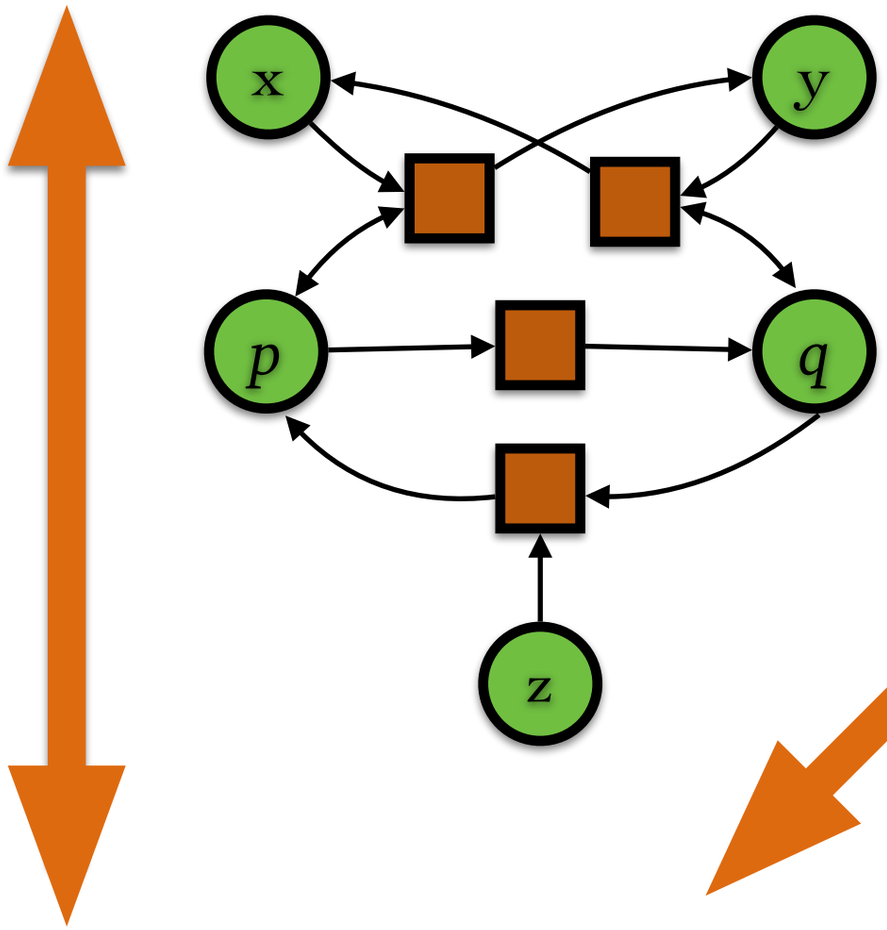

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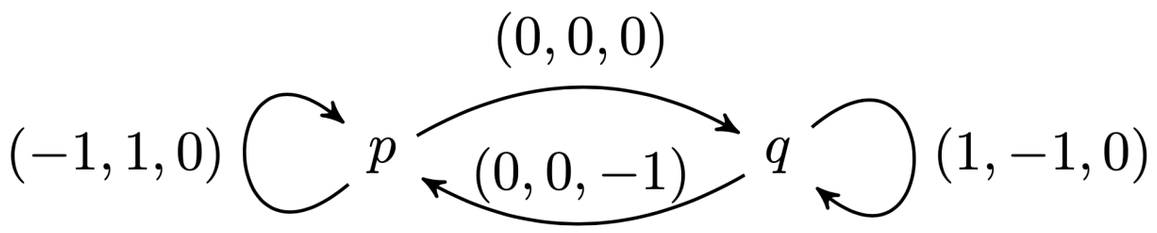

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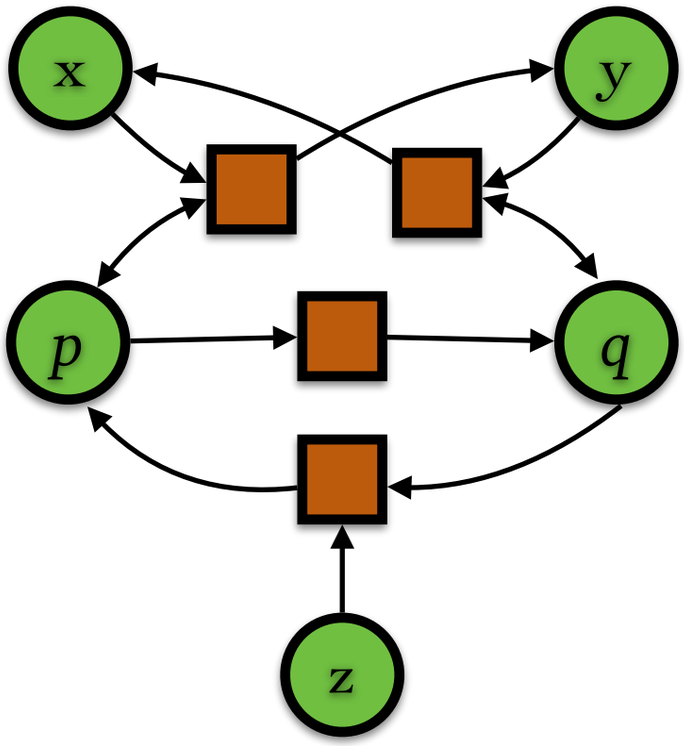

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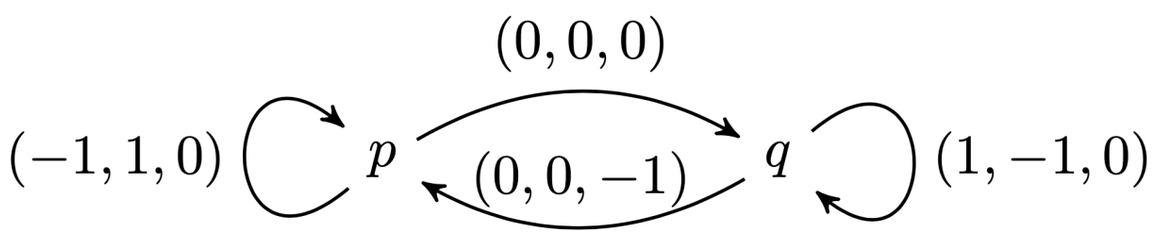

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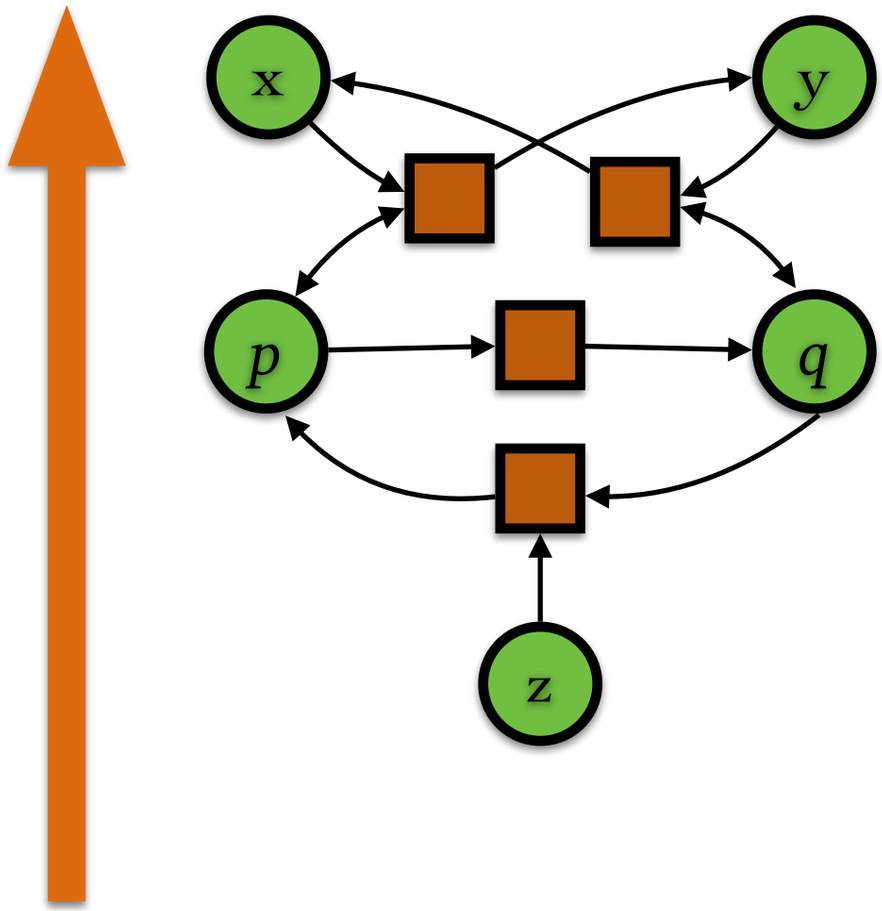

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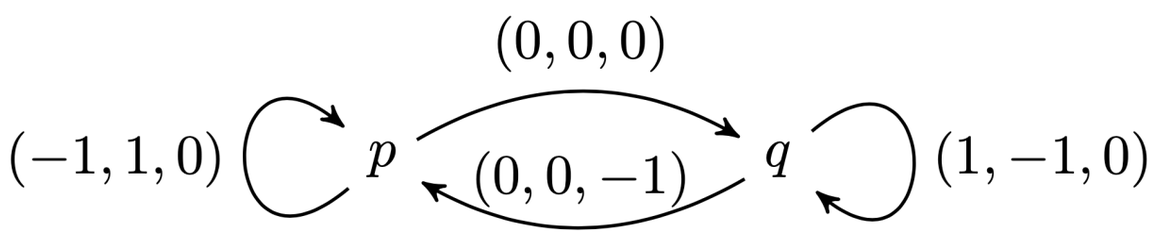

Petri nets $\leftrightarrows$ VASS

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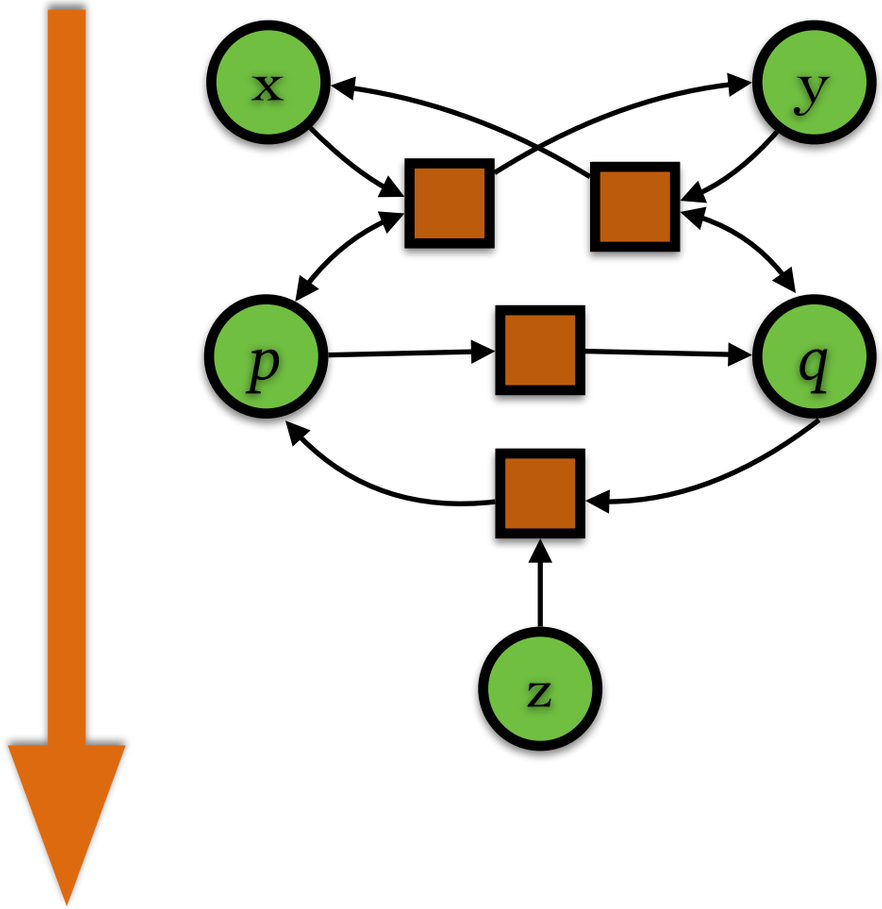
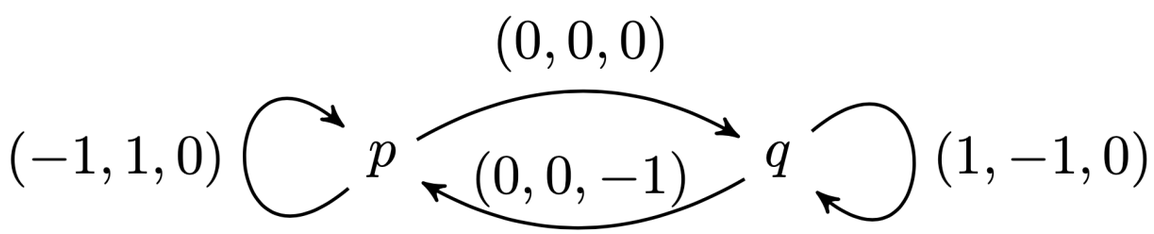
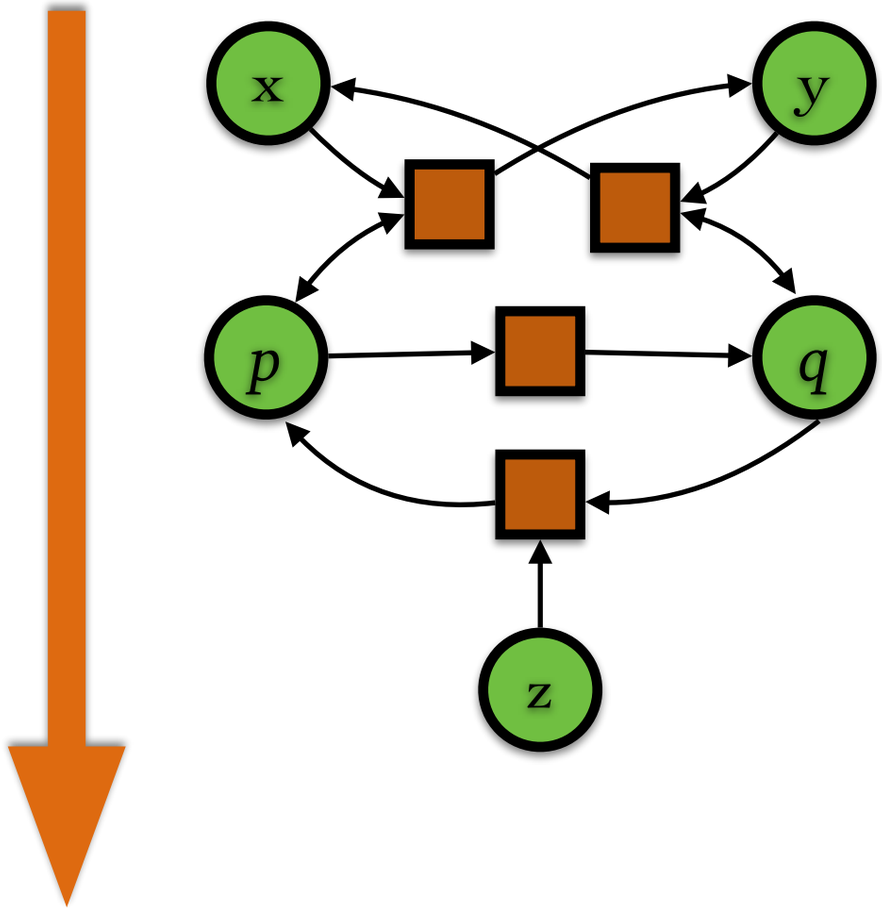


- vector addition systems with states (VASS):

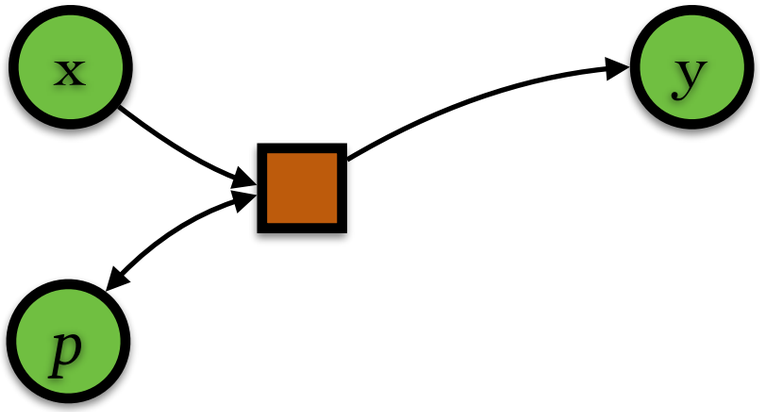


Petri nets $\leftrightarrows$ VASS

- Petri nets:

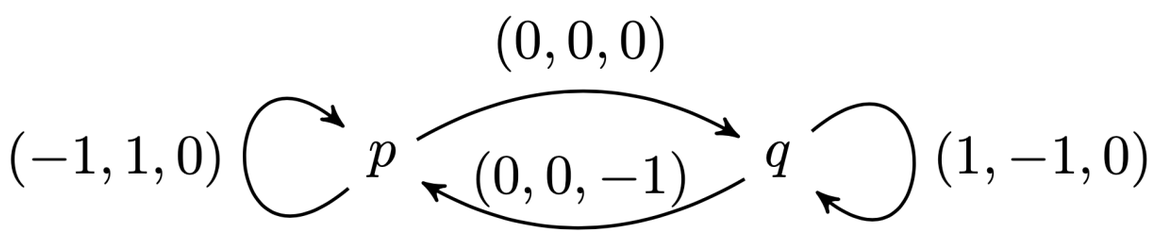


split every transition



into input and output:

- vector addition systems with states (VASS):



Petri nets $\leftrightarrows$ VASS

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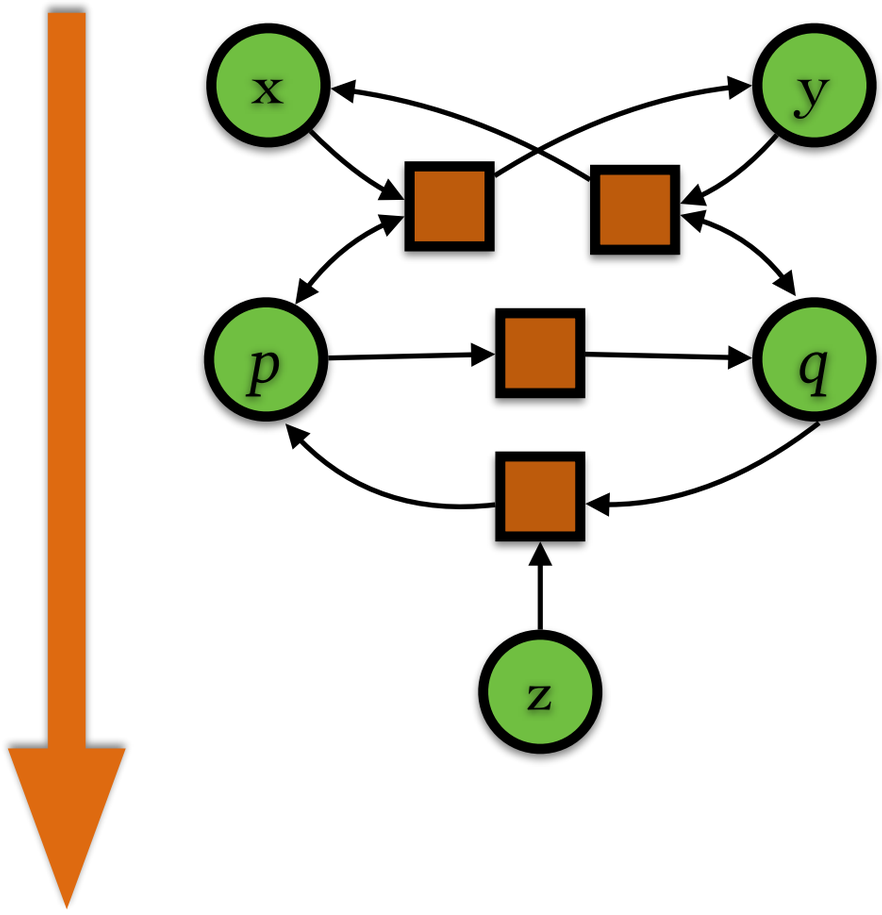
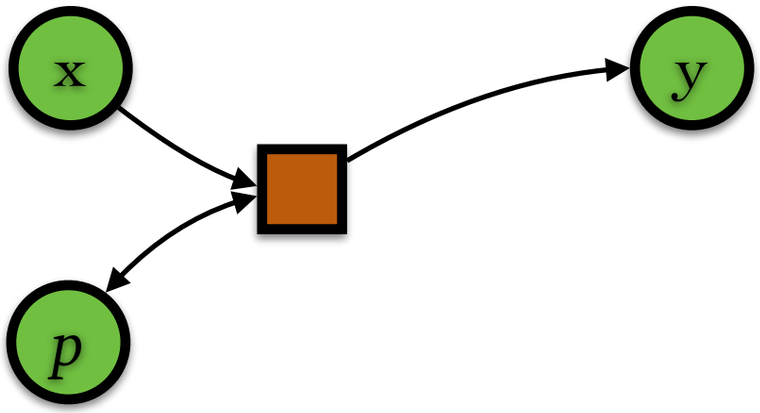
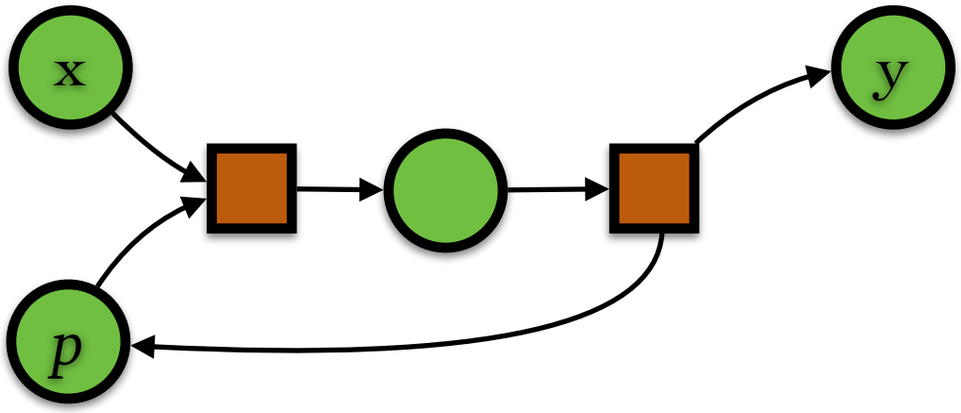


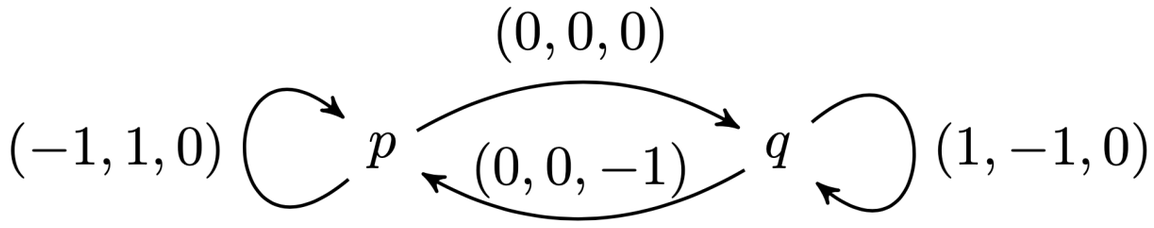
split every transition



into input and output:

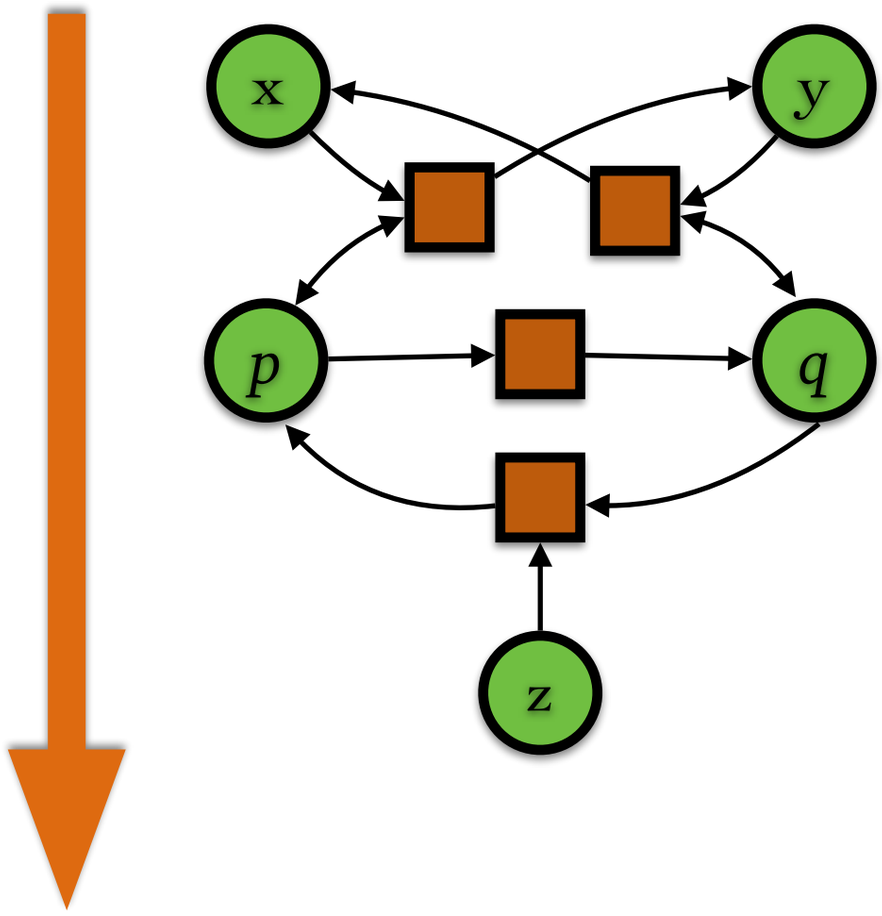


- vector addition systems with states (VASS):

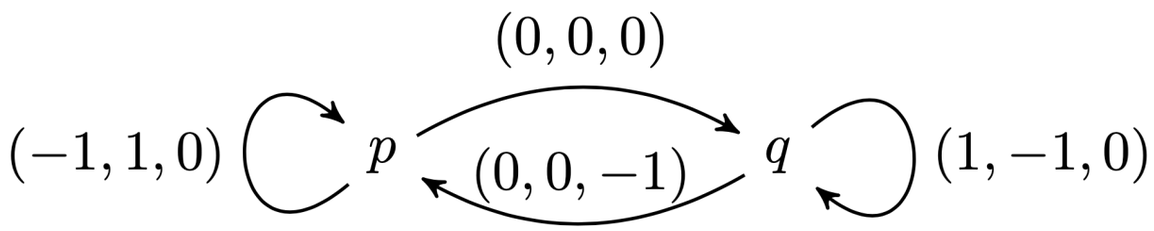


Petri nets $\leftrightarrows$ VASS

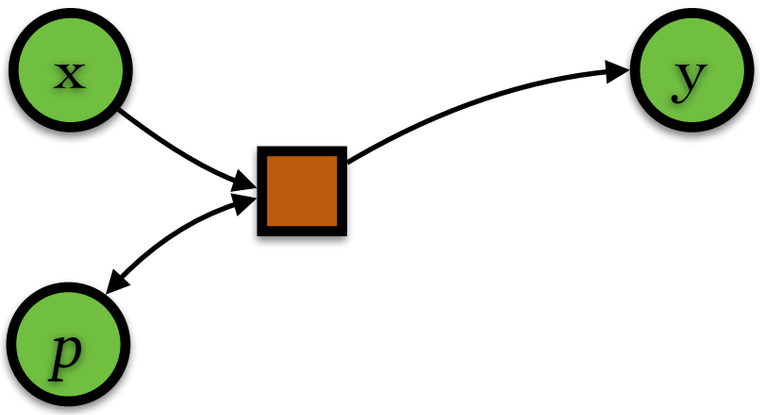
- Petri nets:



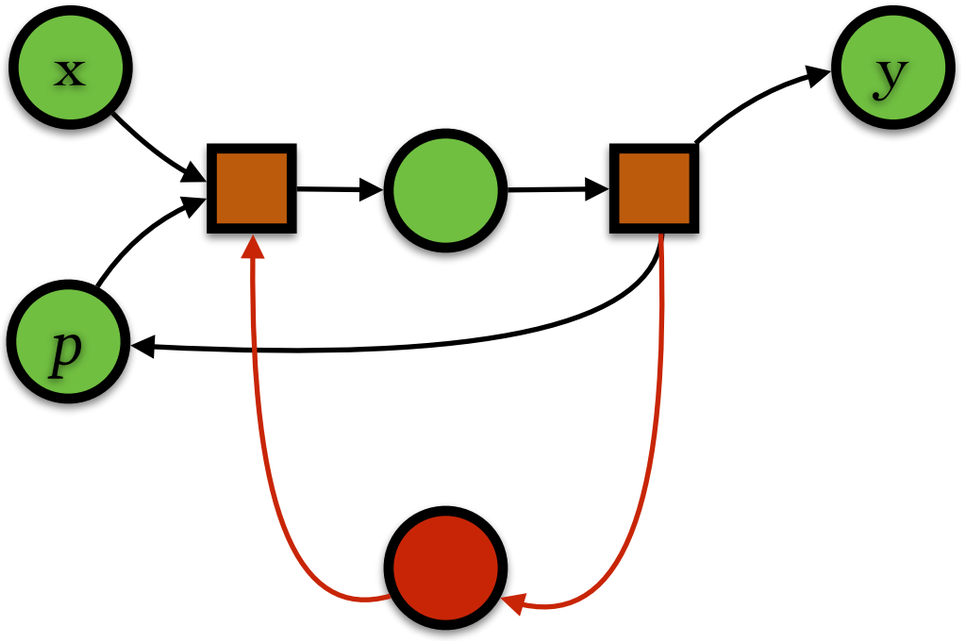
- vector addition systems with states (VASS):



split every transition



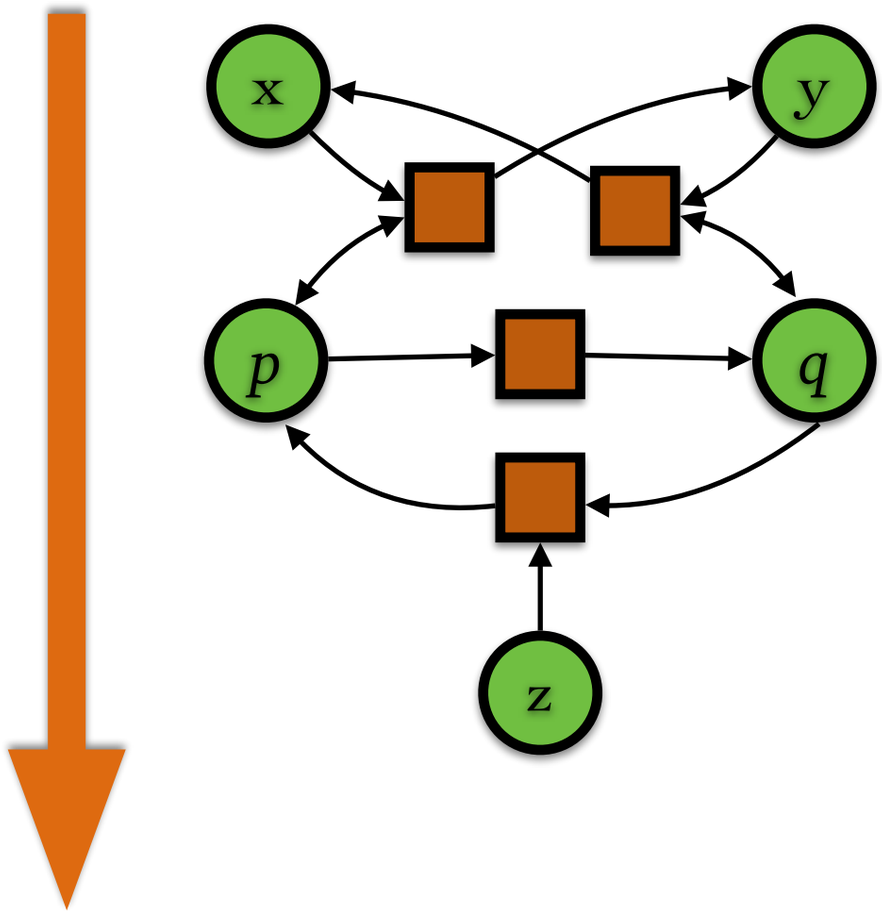
into input and output:



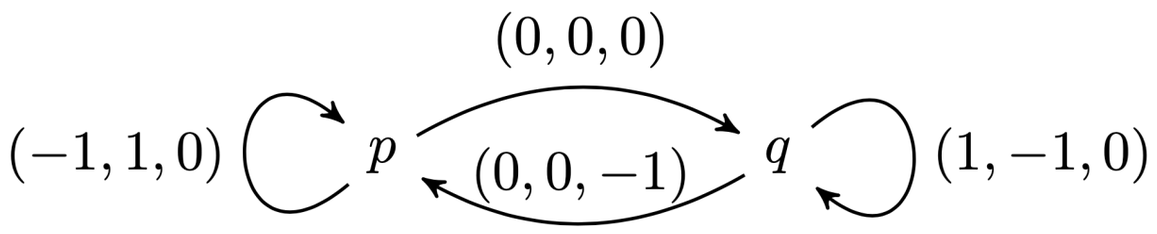
then add one more “global” place

Petri nets $\leftrightarrows$ VASS

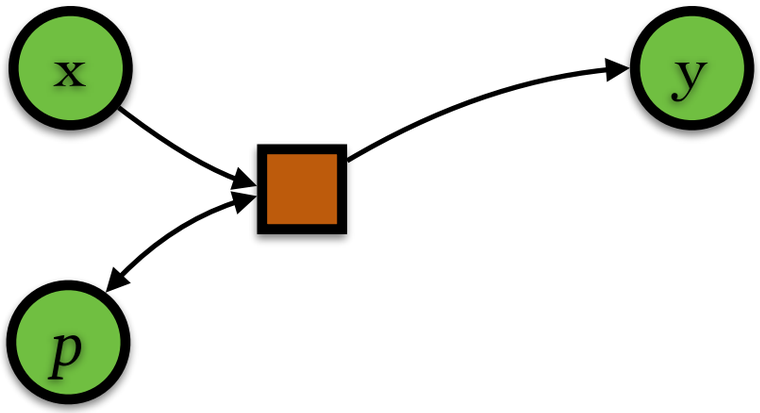
- Petri nets:



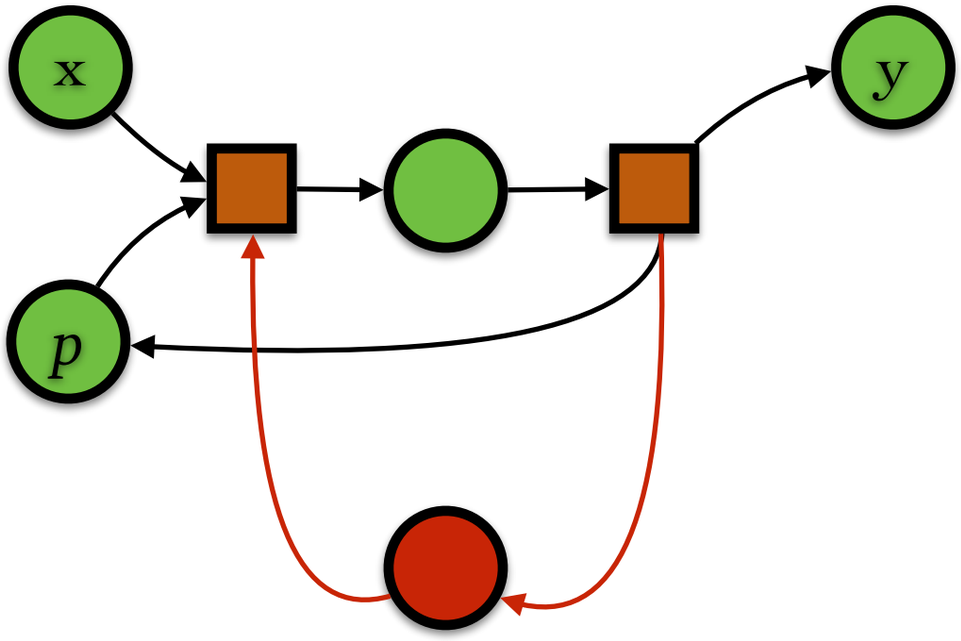
- vector addition systems with states (VASS):



split every transition



into input and output:



then add one more "global" place

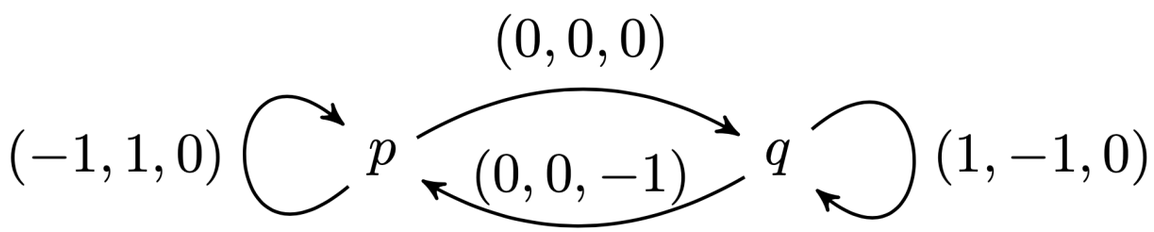
do we actually need  ?

Counter programs $\leftrightarrows$ VASS

- counter programs **without zero-tests**:

```
1:  goto 2 or 10
2:  goto 3 or 5
3:  x −= 1    y += 1
4:  goto 2
5:  goto 6 or 8
6:  x += 1    y −= 1
7:  goto 5
8:  z −= 1
9:  goto 1
10: ...
```

- vector addition systems with states (VASS):



Counter programs $\leftrightarrows$ VASS

- dimension = number of counters
- control states = control locations
- transitions = commands

- counter programs **without zero-tests**:

```
 1: goto 2 or 10
 2: goto 3 or 5
 3: x −= 1    y += 1
 4: goto 2
 5: goto 6 or 8
 6: x += 1    y −= 1
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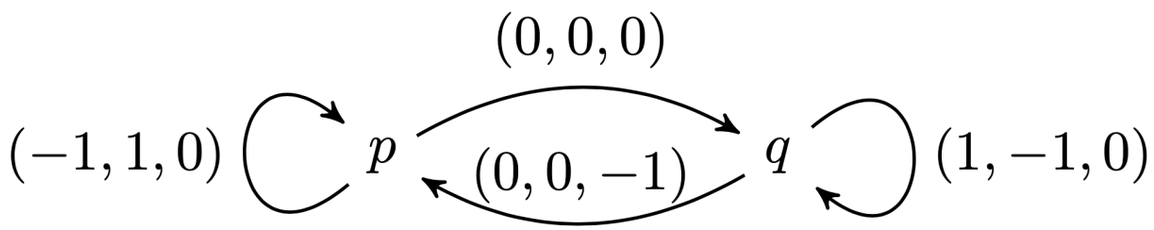
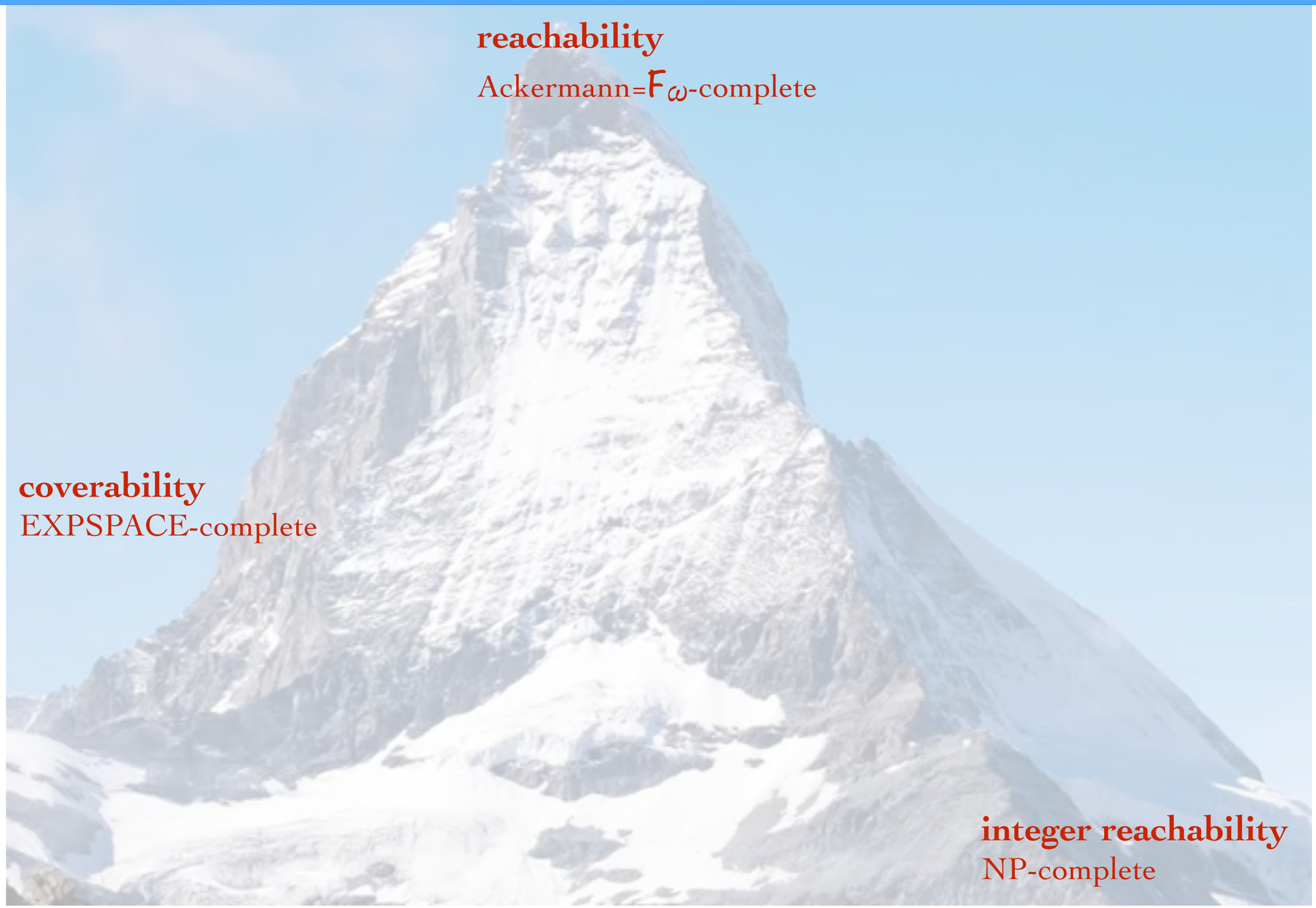
- vector addition systems with states (VASS):



VASS


reachability
Ackermann=F_ω-complete
coverability
EXPSPACE-complete
integer reachability
NP-complete

VASS

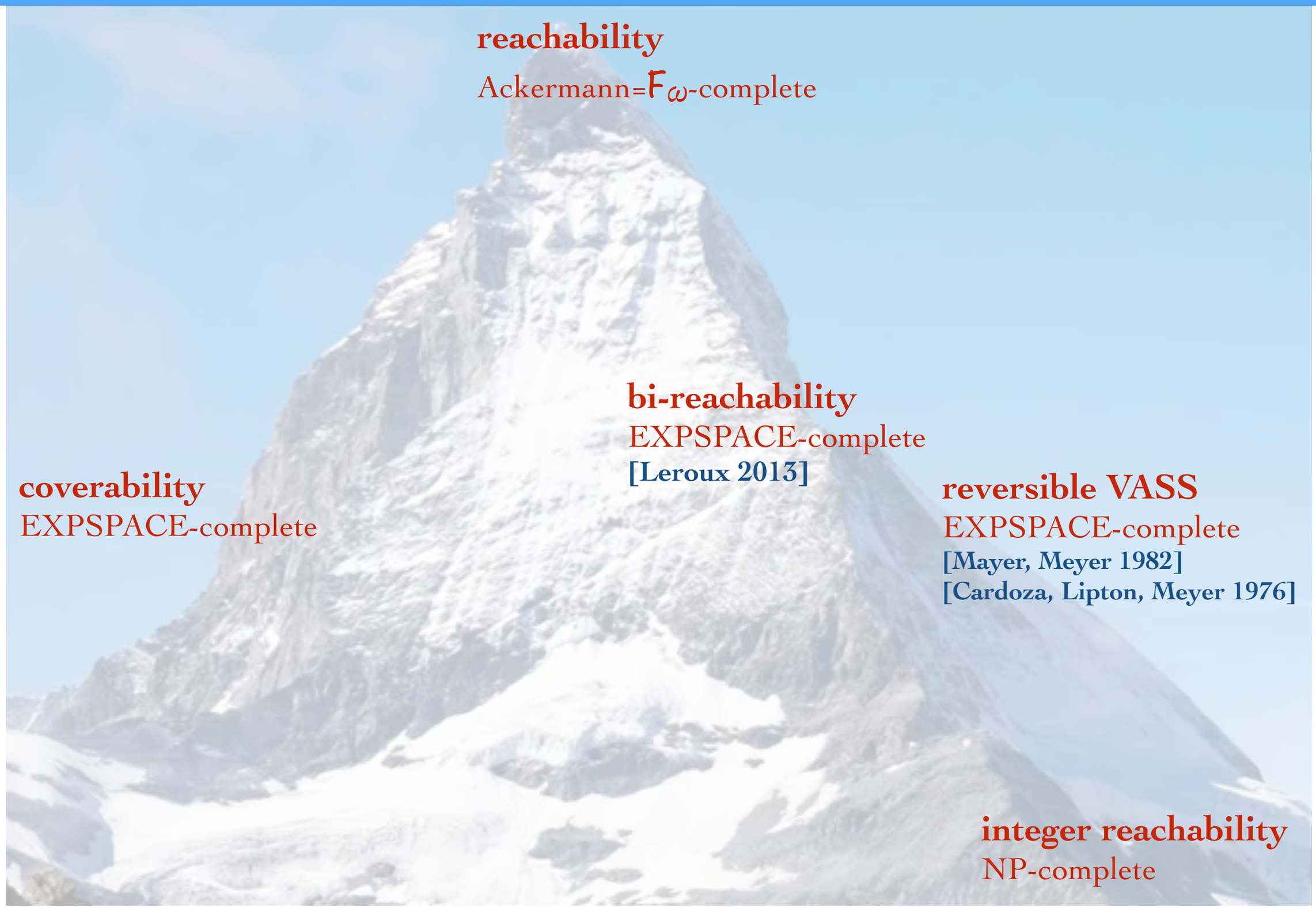
reachability
Ackermann=F_ω-complete

coverability
EXPSPACE-complete

reversible VASS
EXPSPACE-complete
[Mayer, Meyer 1982]
[Cardoza, Lipton, Meyer 1976]

integer reachability
NP-complete

VASS


reachability
Ackermann=F_ω-complete
bi-reachability
EXPSPACE-complete
[Leroux 2013]
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integer reachability
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VASS

reachability
Ackermann=F_ω-complete

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coverability
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dimension 1
NP-complete
[Haase, Kreutzer, Ouaknine, Joel 2009]

integer reachability
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VASS

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dimension 2
PSPACE-complete
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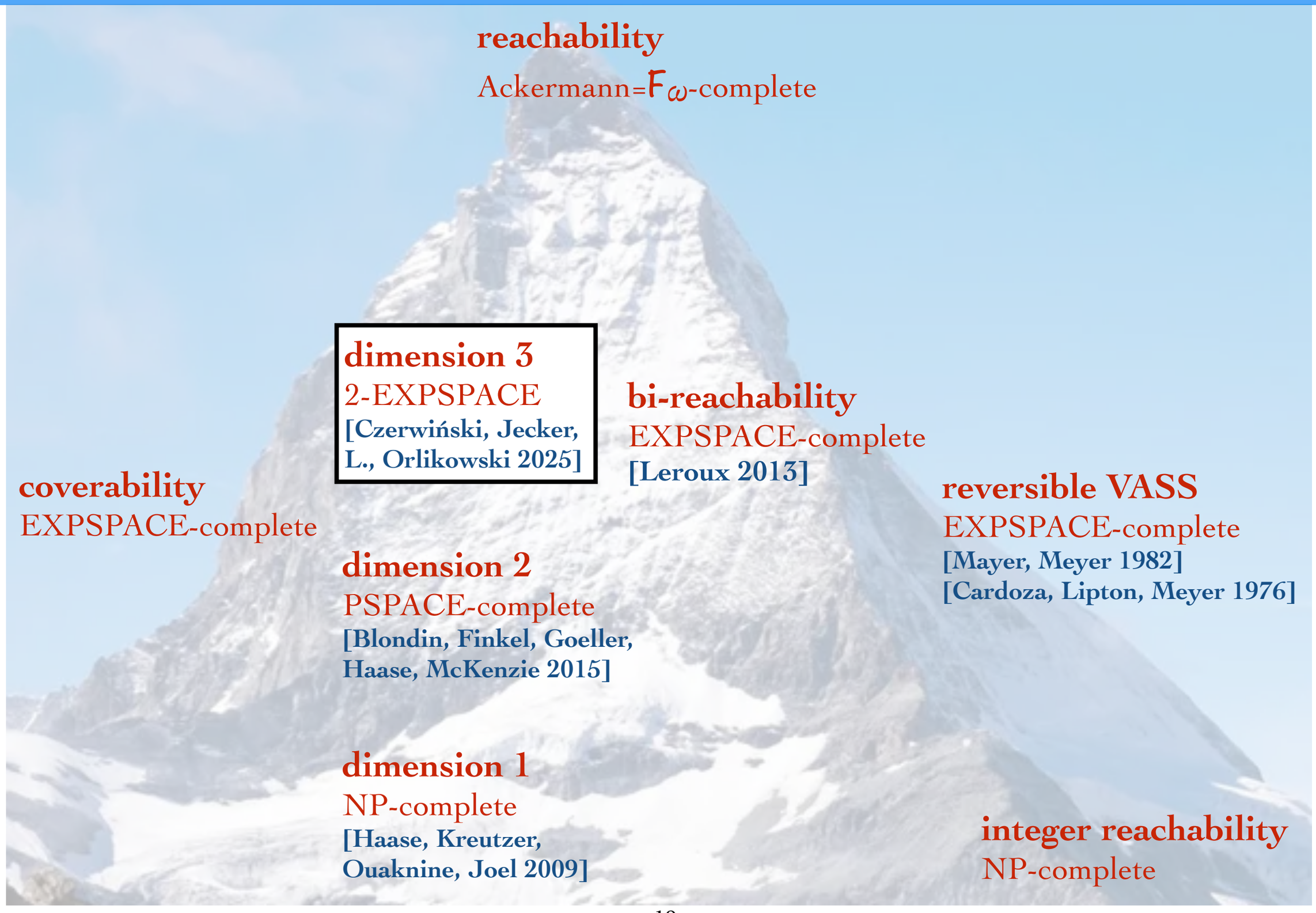
VASS


reachability
Ackermann=F_ω-complete
dimension 3
2-EXPSPACE
[Czerwiński, Jecker, L., Orlikowski 2025]
bi-reachability
EXPSPACE-complete
[Leroux 2013]
coverability
EXPSPACE-complete
reversible VASS
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dimension 2
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NP-complete
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integer reachability
NP-complete

VASS

reachability
Ackermann=F_ω-complete

dimension d
?

dimension 3
2-EXPSPACE
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dimension 1
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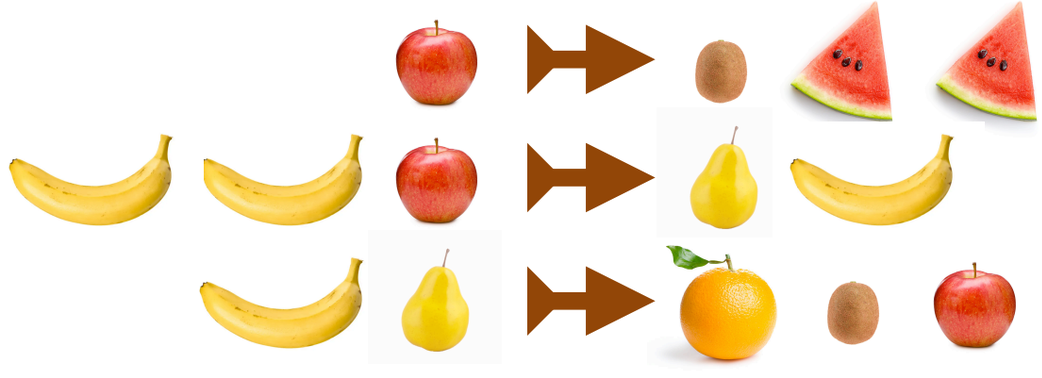
integer reachability
NP-complete

I. Reachability in Petri nets

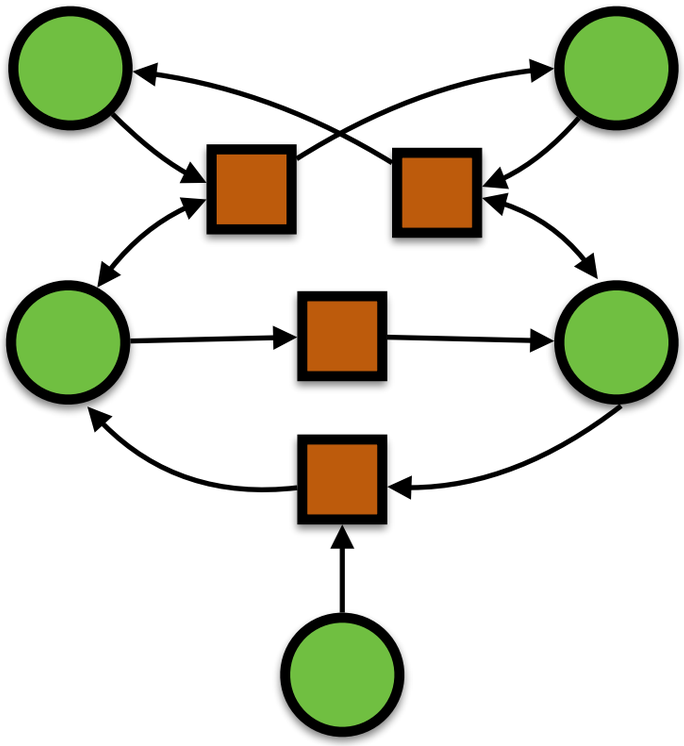
II. Extensions of Petri nets

- **pushdown stack**
- **branching**
- **colors**
- **arbitrary step relations**



Pushdown VASS

- Petri nets:



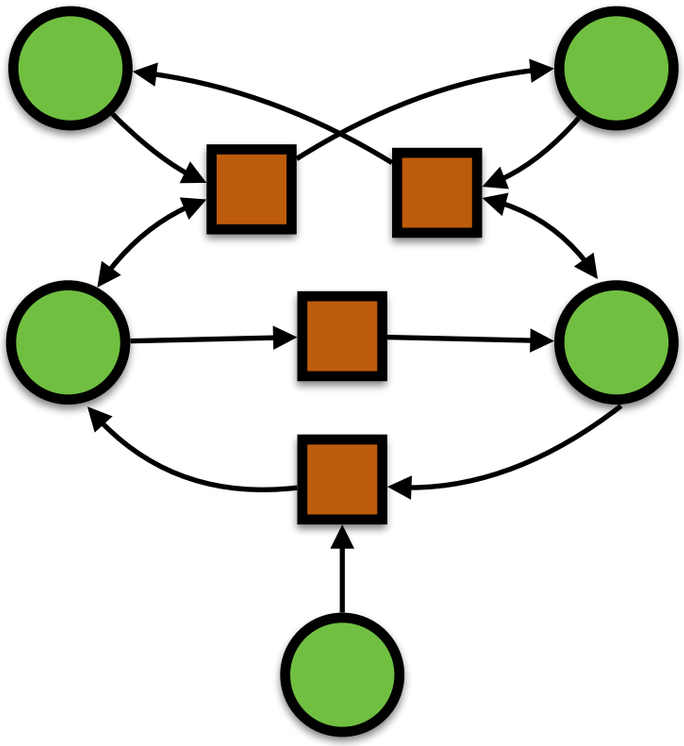
- counter programs **without zero-tests**:

```
1: loop
2:     loop
3:         x −= 1    y += 1
4:     loop
5:         x += 1    y −= 1
6:     z −= 1
```

- vector addition systems with states (VASS):

Pushdown VASS

- Petri nets:



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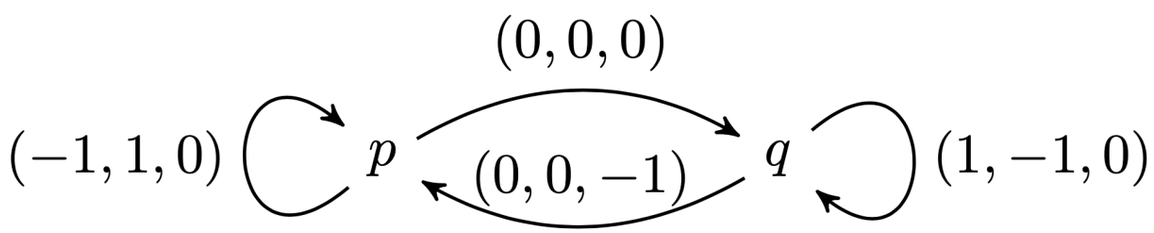
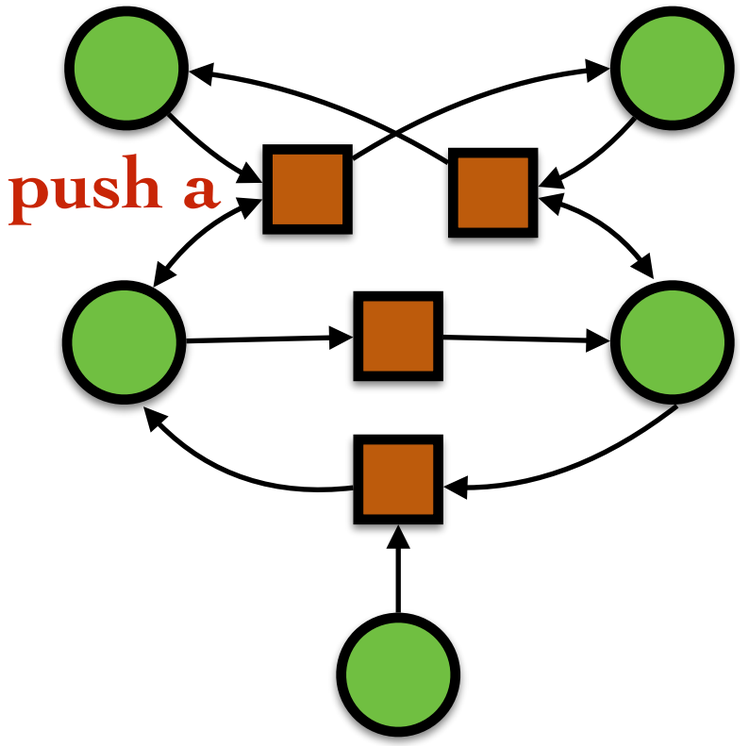
- vector addition systems with states (VASS):



Pushdown VASS

- Petri nets:



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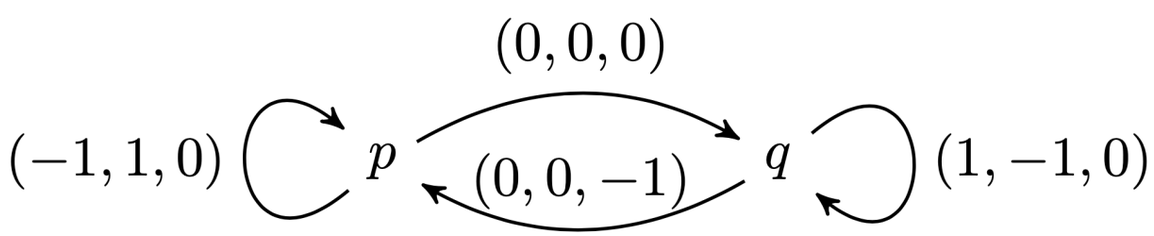
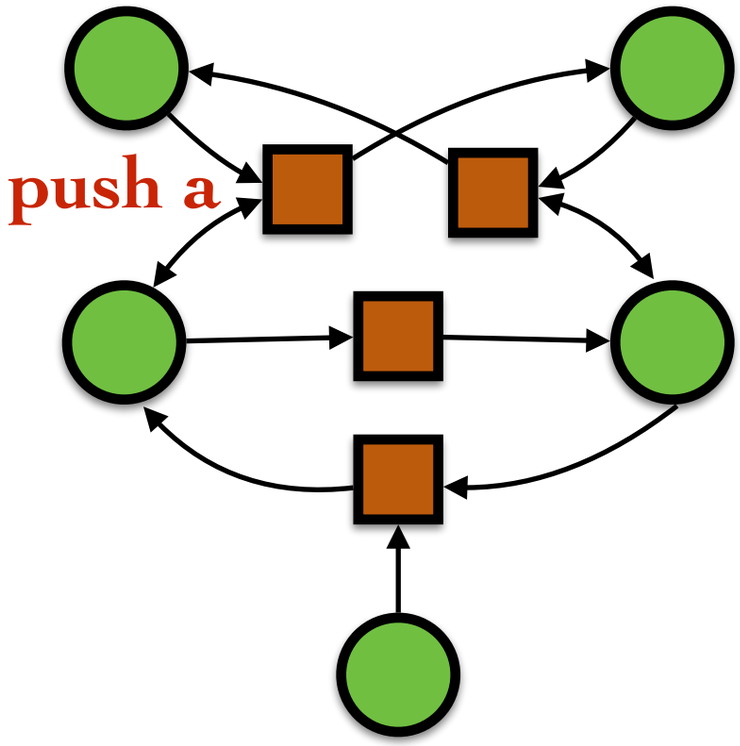


Pushdown VASS

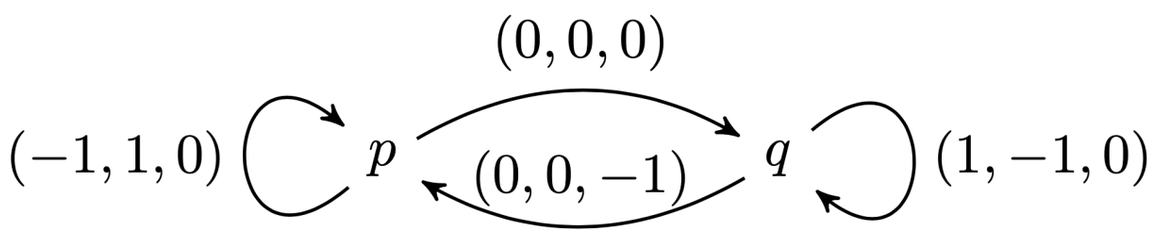
- Petri nets:



- counter programs **without zero-tests**:

```
1: loop
2:     loop  push a
3:         x −= 1    y += 1
4:     loop
5:         x += 1    y −= 1
6:     z −= 1
```

- vector addition systems with states (VASS):



Pushdown VASS

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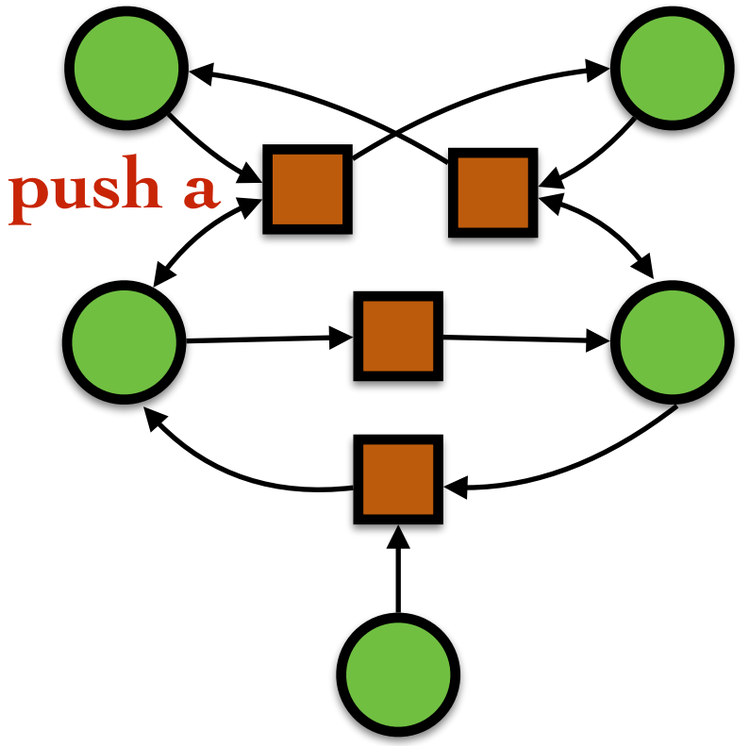

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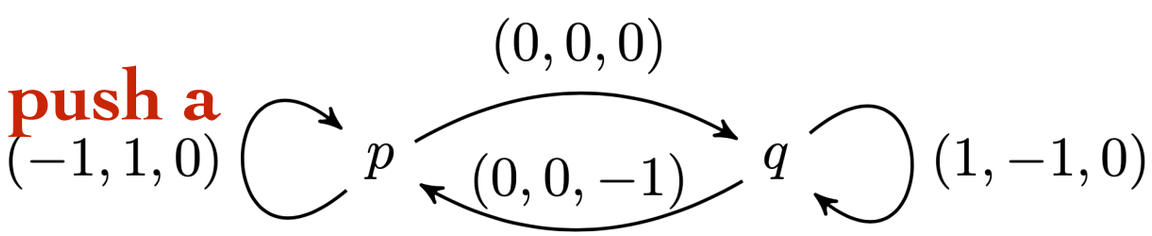
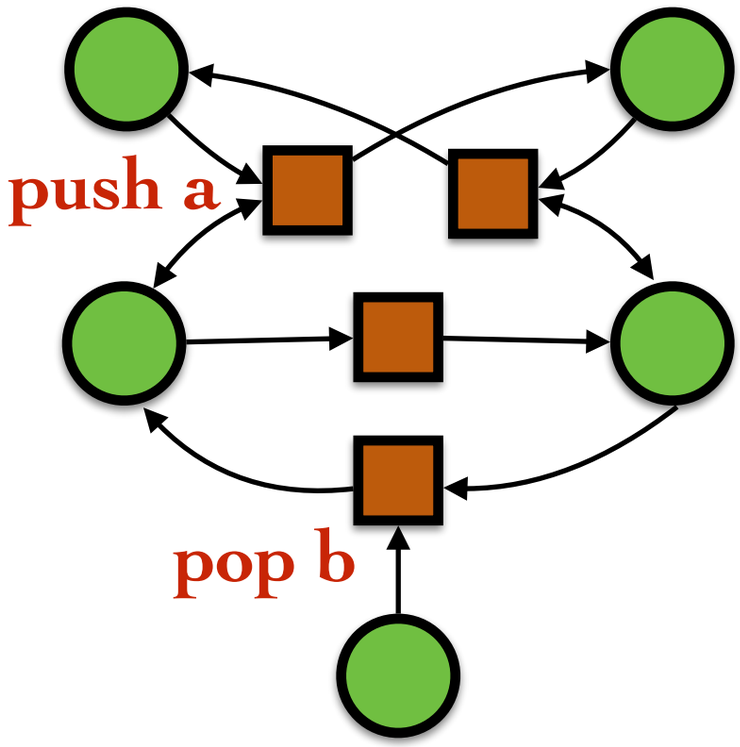
2:     loop  push a
3:         x −= 1    y += 1
4:     loop
5:         x += 1    y −= 1
6:     z −= 1
```

- vector addition systems with states (VASS):



Pushdown VASS

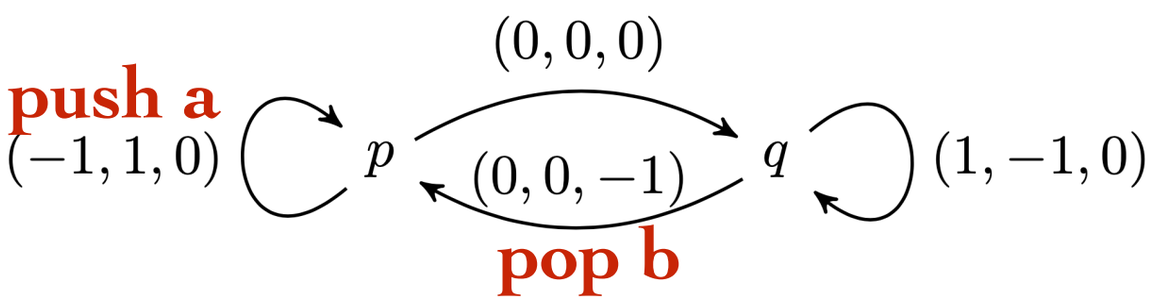
- Petri nets:



- counter programs **without zero-tests**:

```
1: loop
2:     loop  push a
3:         x −= 1    y += 1
4:     loop
5:         x += 1    y −= 1
6:     z −= 1  pop b
```

- vector addition systems with states (VASS):



Pushdown VASS reachability

Equivalent formulations:

Pushdown VASS reachability

Equivalent formulations:

- VASS reachability controlled by a context-free language

Pushdown VASS reachability

Equivalent formulations:

- VASS reachability controlled by a context-free language
- Context-free language $\cap$ VASS language $\neq \emptyset$

Pushdown VASS reachability

Equivalent formulations:

- VASS reachability controlled by a context-free language
- Context-free language $\cap$ VASS language $\neq \emptyset$
- Context-free language $\cap$ Commutative context-free language $\neq \emptyset$

VASS

reachability
Ackermann=F_ω-complete

dimension d
?

dimension 3
2-EXPSPACE
[Czerwiński, Jecker, L., Orlikowski 2025]

bi-reachability
EXPSPACE-complete
[Leroux 2013]

coverability
EXPSPACE-complete

reversible VASS
EXPSPACE-complete
[Mayer, Meyer 1982]
[Cardoza, Lipton, Meyer 1976]

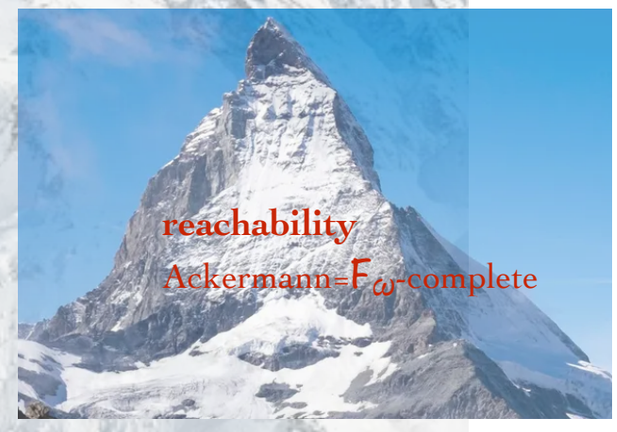
dimension 2
PSPACE-complete
[Blondin, Finkel, Goeller, Haase, McKenzie 2015]

dimension 1
NP-complete
[Haase, Kreutzer, Ouaknine, Joel 2009]

integer reachability
NP-complete

Pushdown VASS

reachability



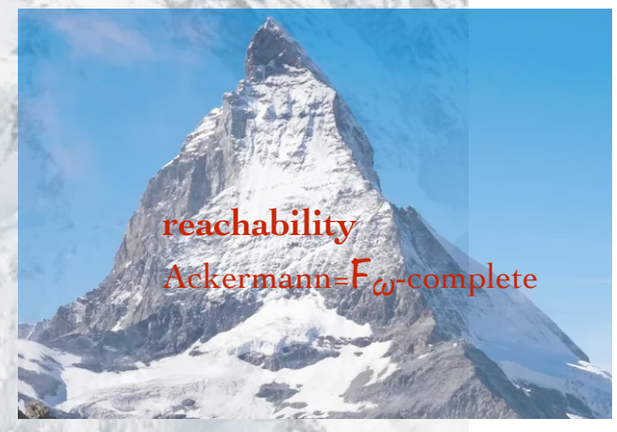
coverability - dimension 1
EXPSPACE
[Leroux, Sutre, Totzke 2015]

integer reachability
NP-complete
[Hague, Widjaja Lin 2011]

Pushdown VASS

reachability

finite-index PVASS
decidable
[Fauozi, Ganty 2011]



coverability - dimension 1
EXPSPACE
[Leroux, Sutre, Totzke 2015]

integer reachability
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Pushdown VASS

reachability

VASS with nested 0-tests ⟷ **finite-index PVASS**

F_ω-complete

[Reinhardt 2008]

[Bonnet 2011]

[Czerwiński, L, Orlikowski 2025]

decidable

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coverability - dimension 1

EXPSPACE

[Leroux, Sutre, Totzke 2015]

integer reachability

NP-complete

[Hague, Widjaja Lin 2011]

Pushdown VASS

reachability

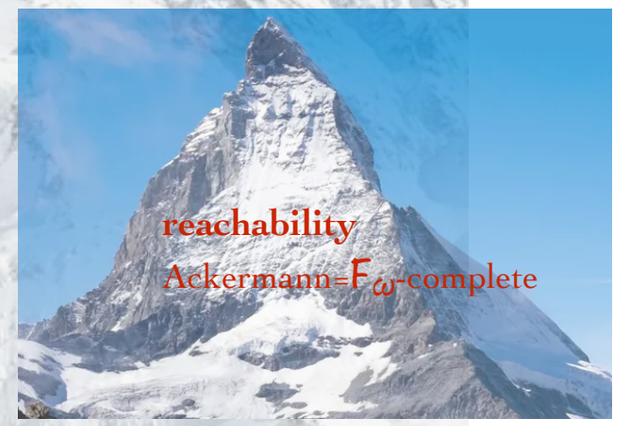
VASS with nested 0-tests ⟷ **finite-index PVASS** ⟷ **reversible PVASS**

F_ω-complete — decidable — F_ω

[Reinhardt 2008] [Fauozi, Ganty 2011] [Ganardi, Majumdar, Zetzsche 2022]

[Bonnet 2011]

[Czerwiński, L, Orlikowski 2025]



coverability - dimension 1

EXPSPACE

[Leroux, Sutre, Totzke 2015]

integer reachability

NP-complete

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Pushdown VASS

reachability

VASS with nested 0-tests ⟷ **finite-index PVASS**

F_ω-complete

[Reinhardt 2008]

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reversible PVASS

F_ω

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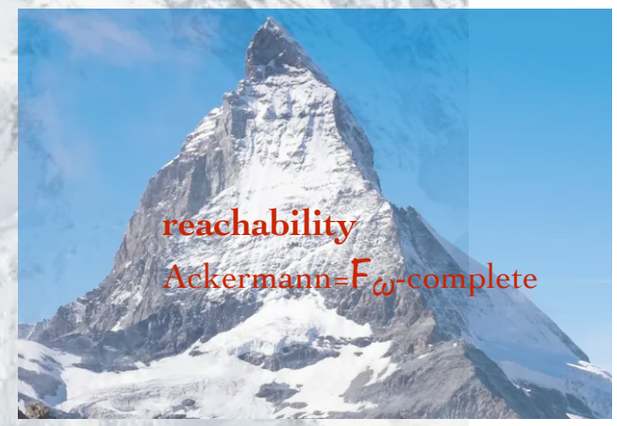

reachability - dimension 1

F_ω

[Biziere, Czerwiński 2025]

coverability - dimension 1

EXPSPACE

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integer reachability

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Pushdown VASS

reachability

VASS with nested 0-tests ⟷ **finite-index PVASS** ⟷ **reversible PVASS**

F_ω-complete

[Reinhardt 2008]
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decidable

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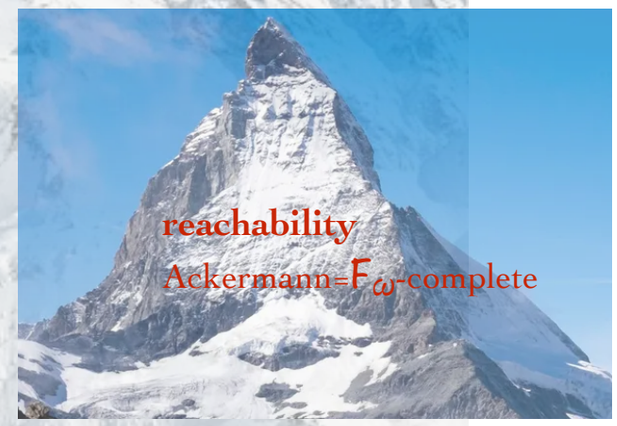
F_ω

[Ganardi, Majumdar, Zetzsche 2022]

coverability - dimension 2

?



reachability - dimension 1

F_ω

[Biziere, Czerwiński 2025]

coverability - dimension 1

EXPSPACE

[Leroux, Sutre, Totzke 2015]

integer reachability

NP-complete

[Hague, Widjaja Lin 2011]

Pushdown VASS

coverability ⟷ reachability

VASS with nested 0-tests ⟷ **finite-index PVASS** ⟷ **reversible PVASS**

F_ω-complete — decidable — F_ω

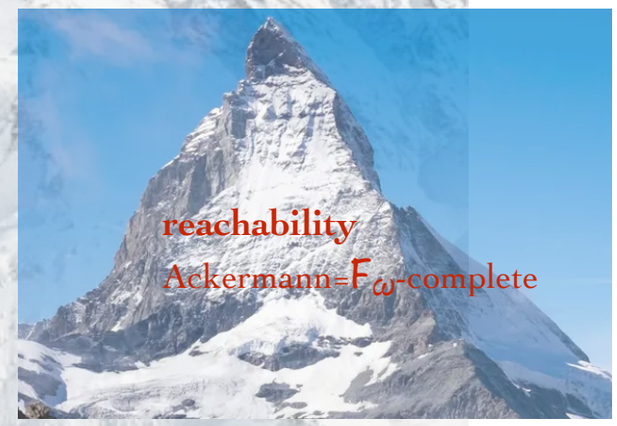
[Reinhardt 2008] — [Fauozi, Ganty 2011] — [Ganardi, Majumdar, Zetzsche 2022]

[Bonnet 2011]

[Czerwiński, L, Orlikowski 2025]

coverability - dimension 2

?



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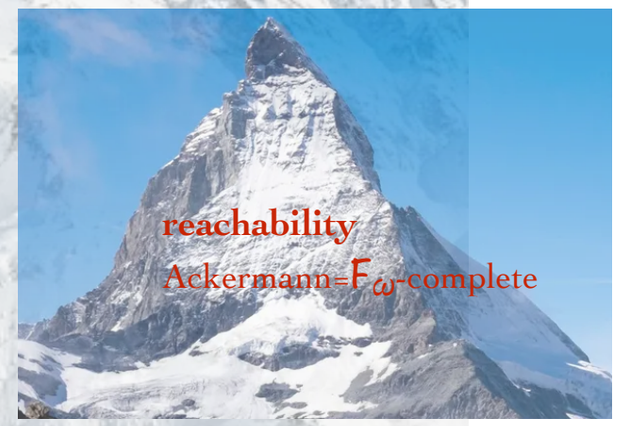
coverability - dimension 2

?



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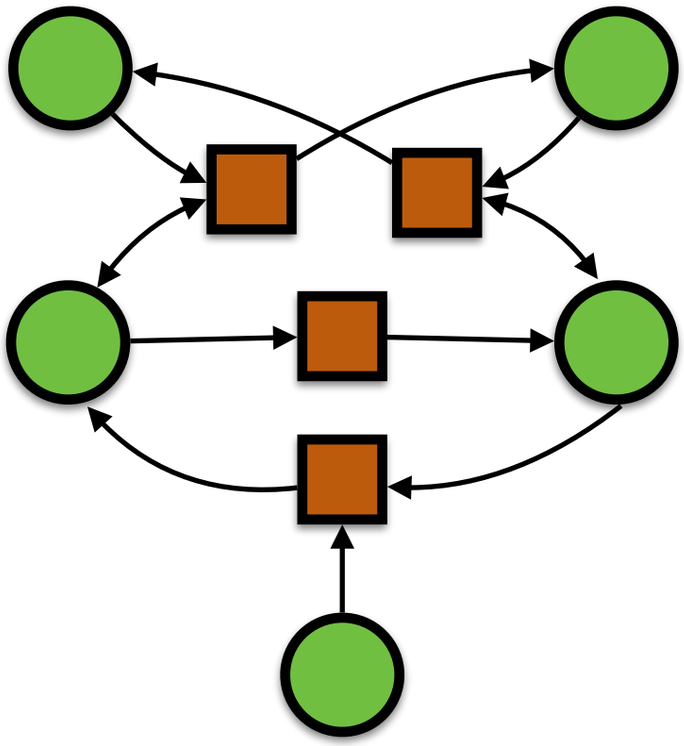
Branching VASS



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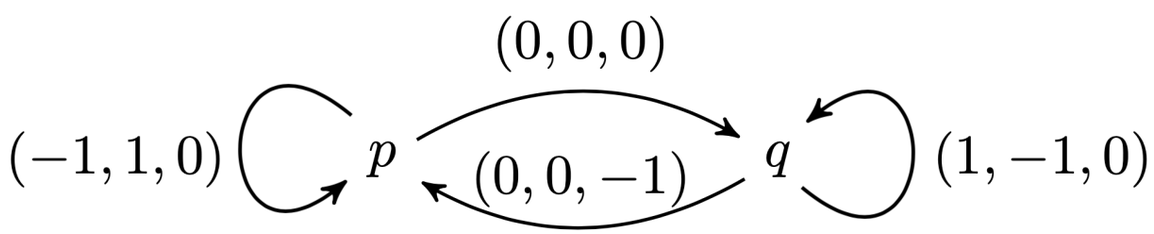

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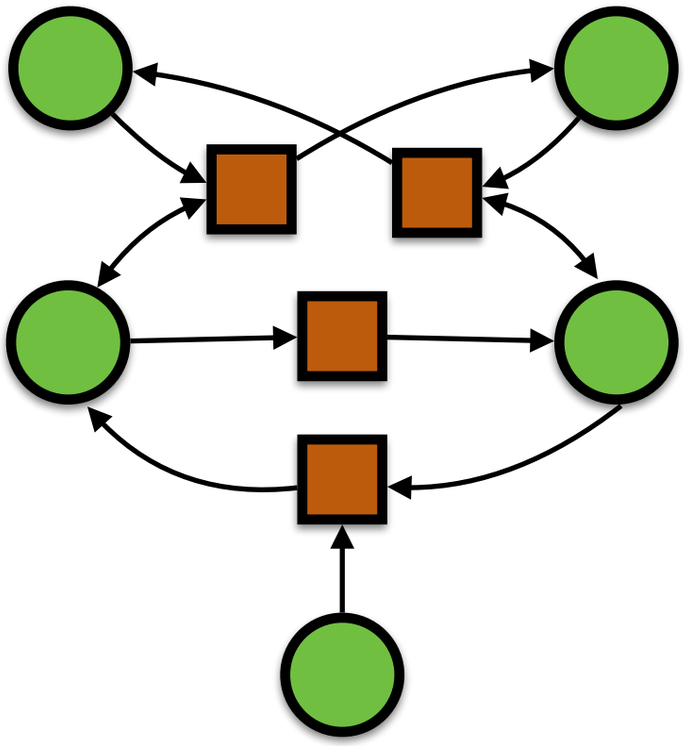

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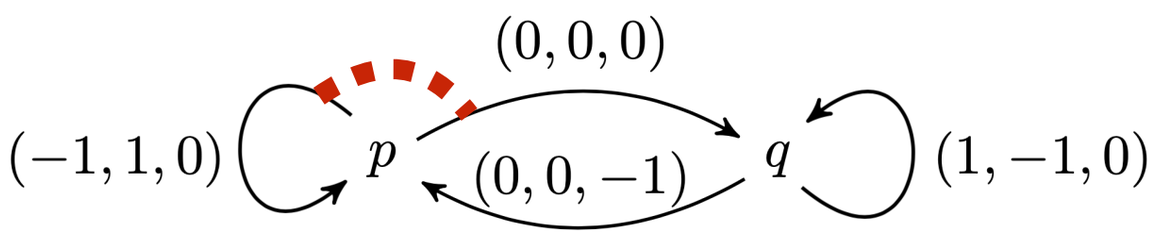

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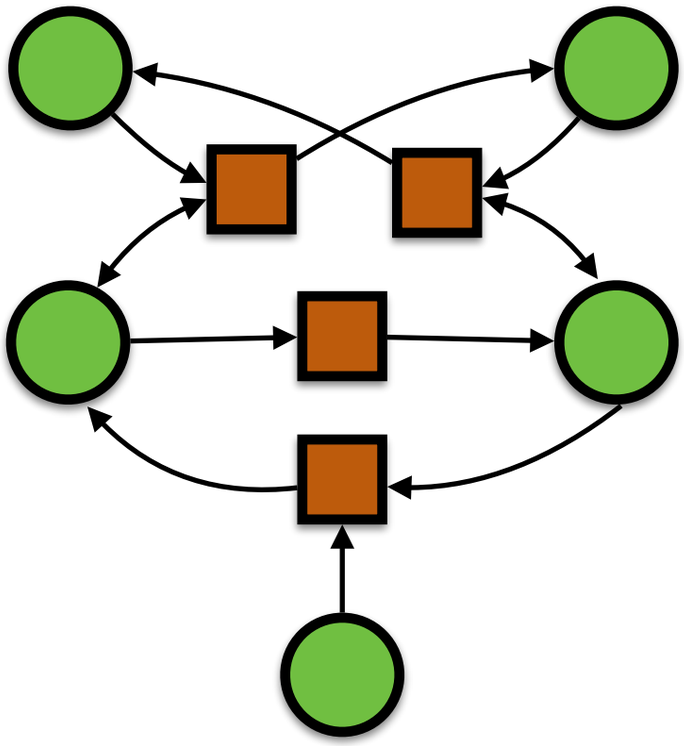

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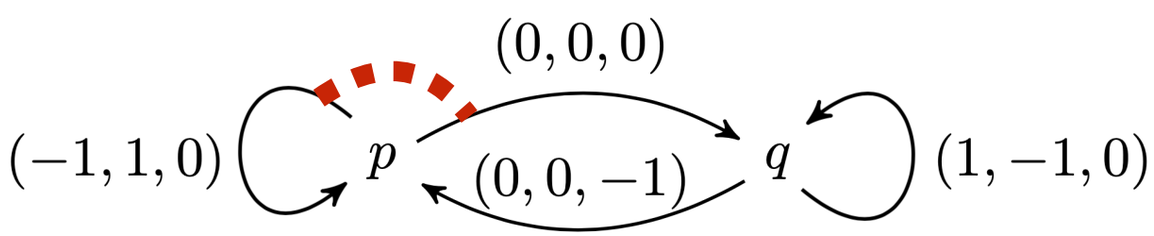

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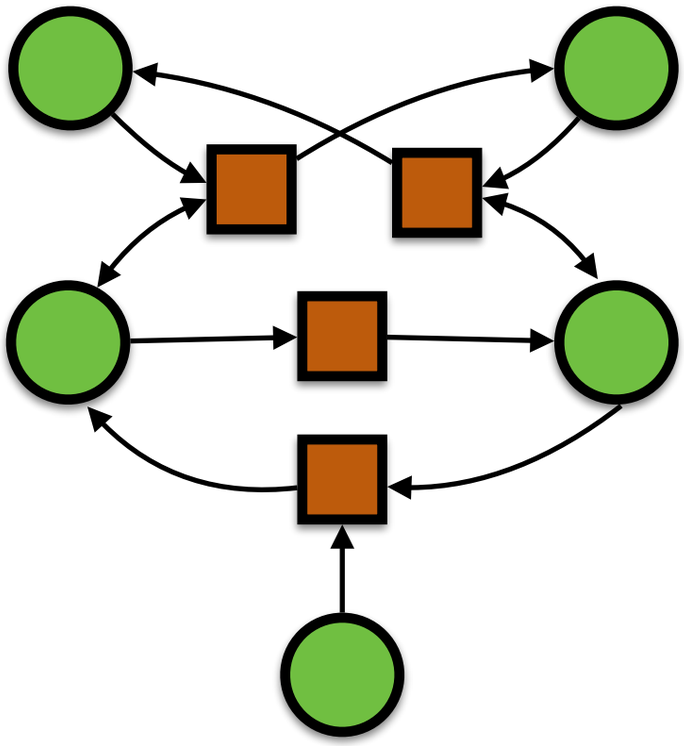

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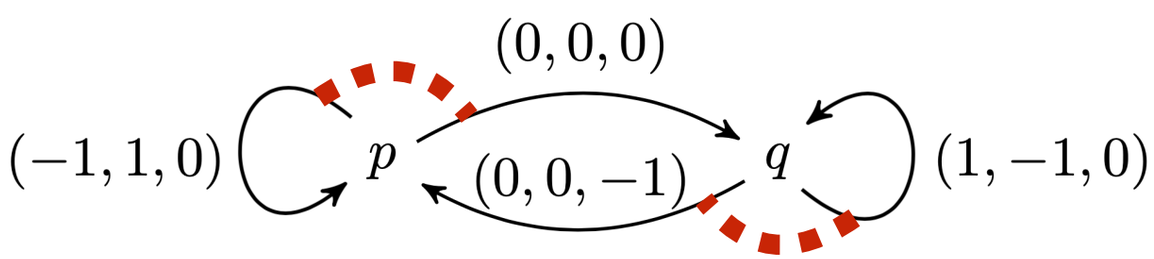

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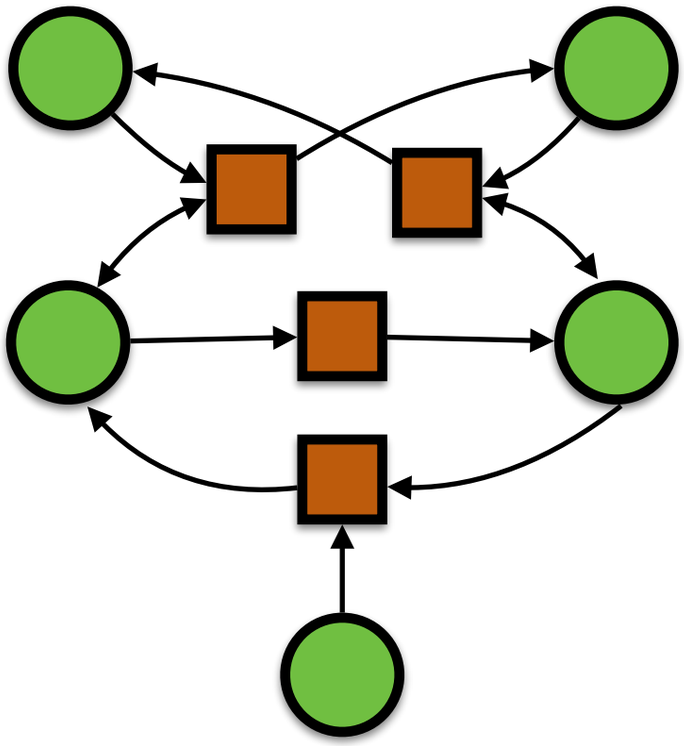

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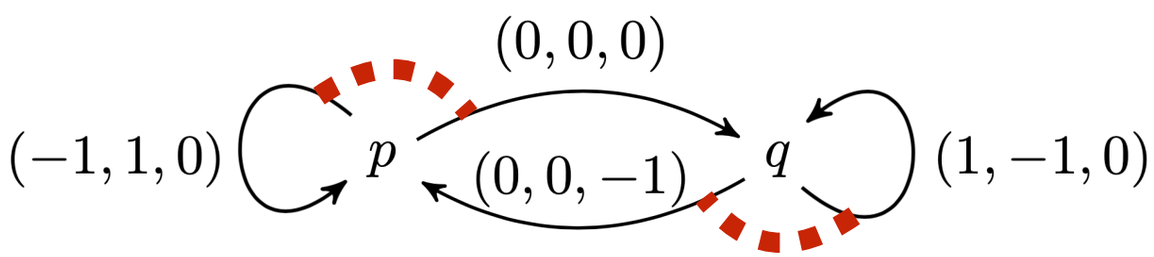
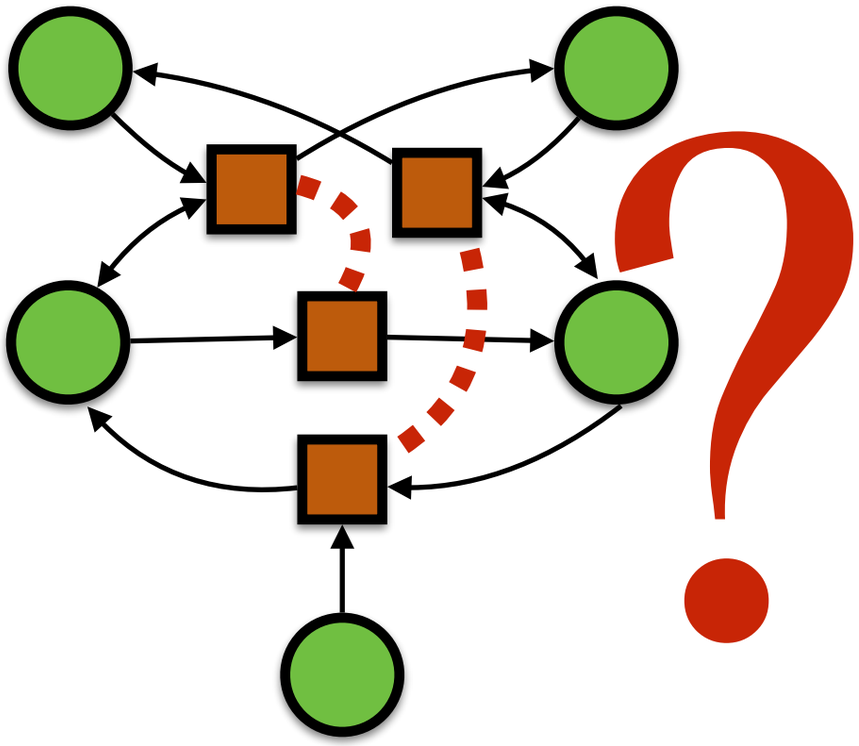


Branching VASS



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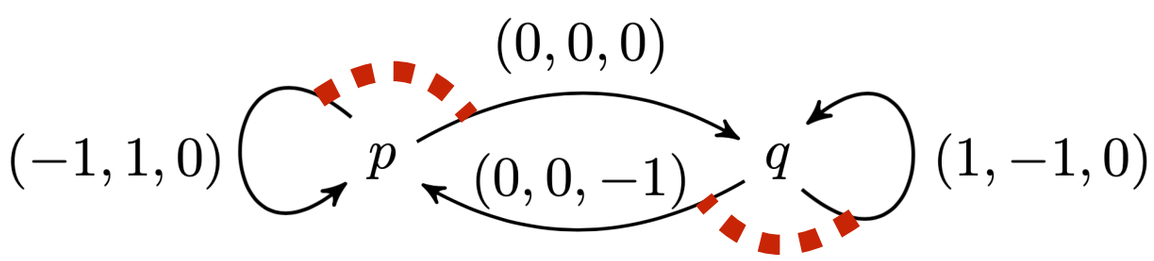

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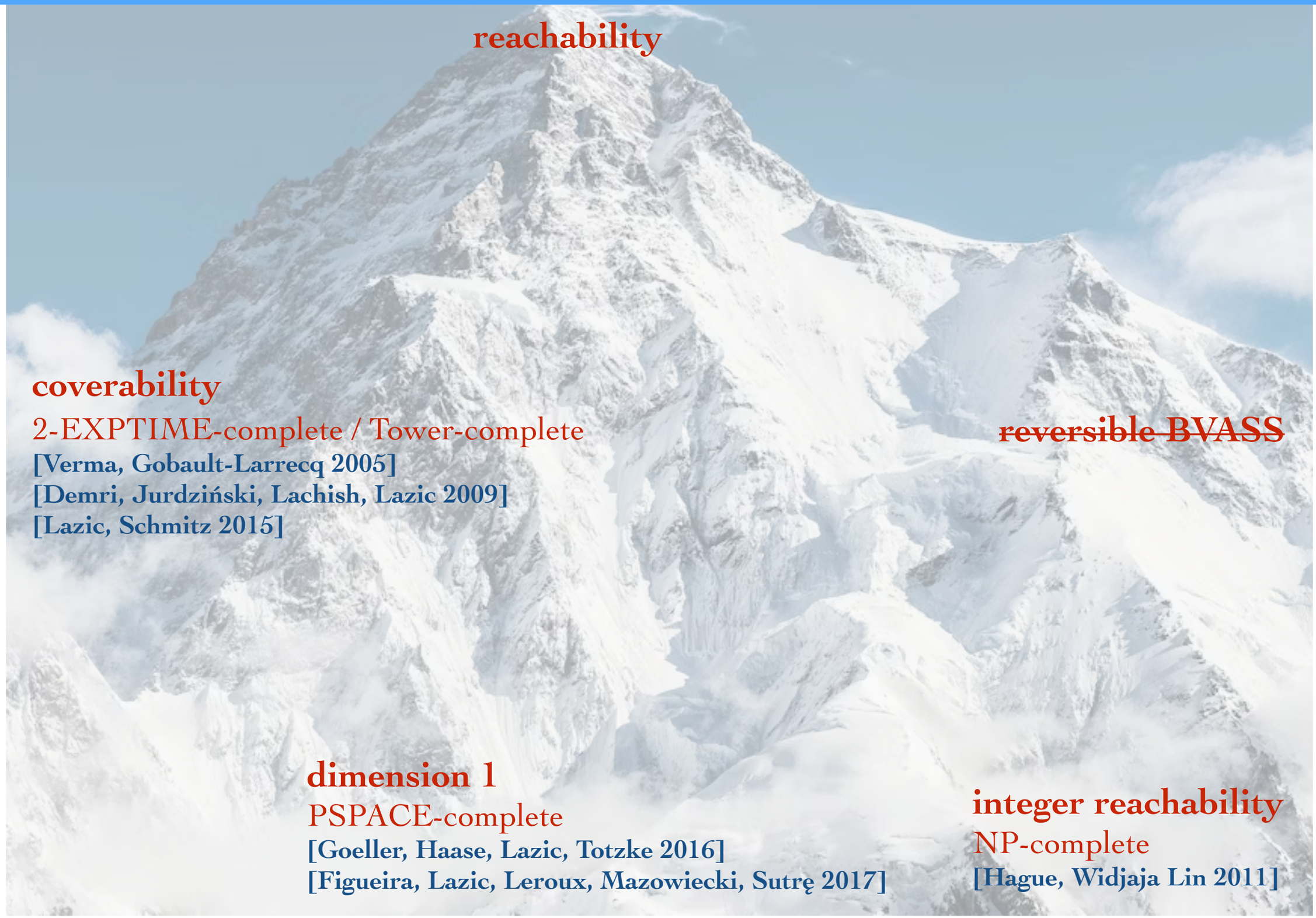

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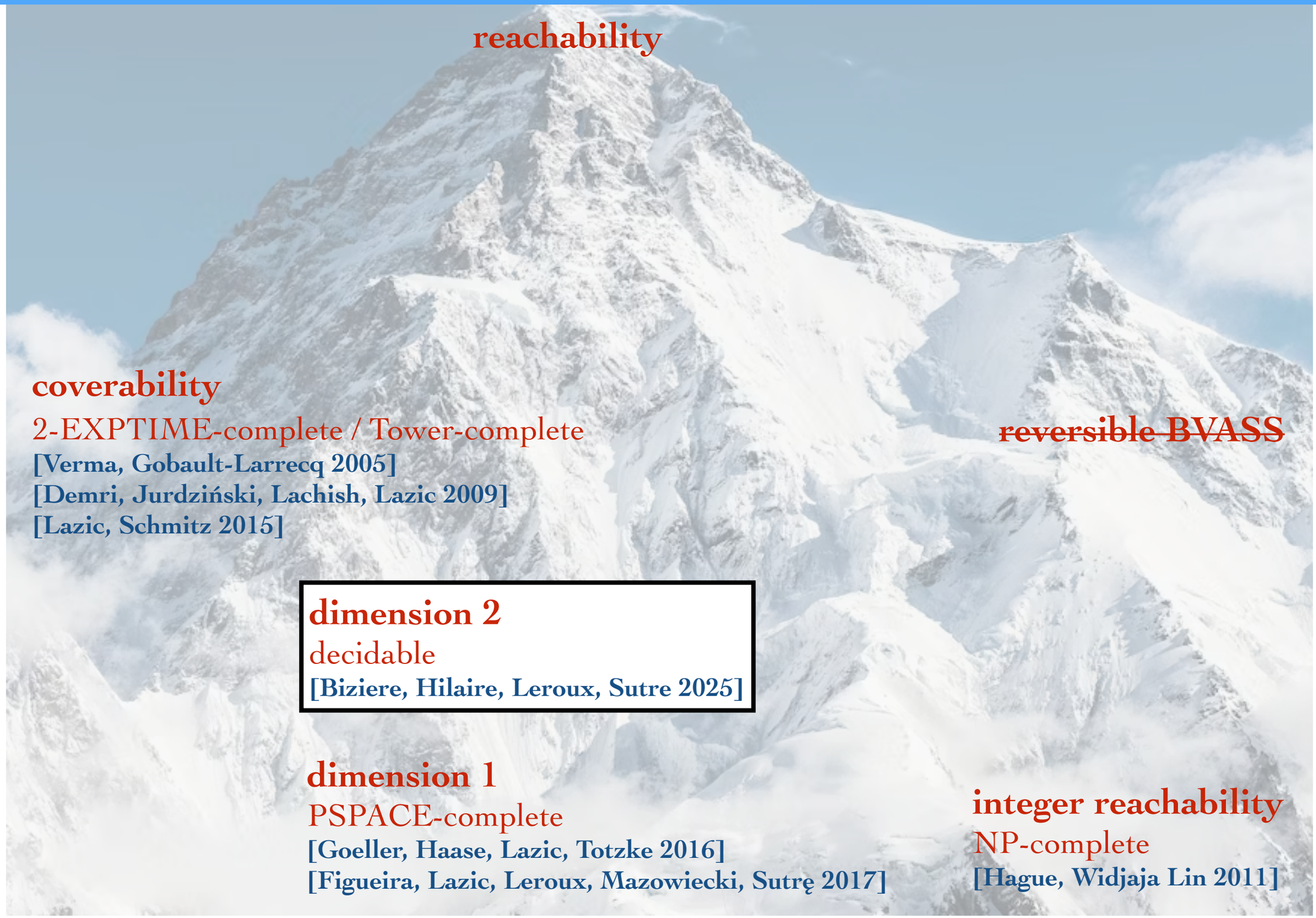

reachability

coverability

2-EXPTIME-complete / Tower-complete

[Verma, Gobault-Larrecq 2005]

[Demri, Jurdziński, Lachish, Lazic 2009]

[Lazic, Schmitz 2015]

~~reversible BVASS~~

dimension 2

decidable

[Biziere, Hilaire, Leroux, Sutre 2025]

dimension 1

PSPACE-complete

[Goeller, Haase, Lazic, Totzke 2016]

[Figueira, Lazic, Leroux, Mazowiecki, Sutrę 2017]

integer reachability

NP-complete

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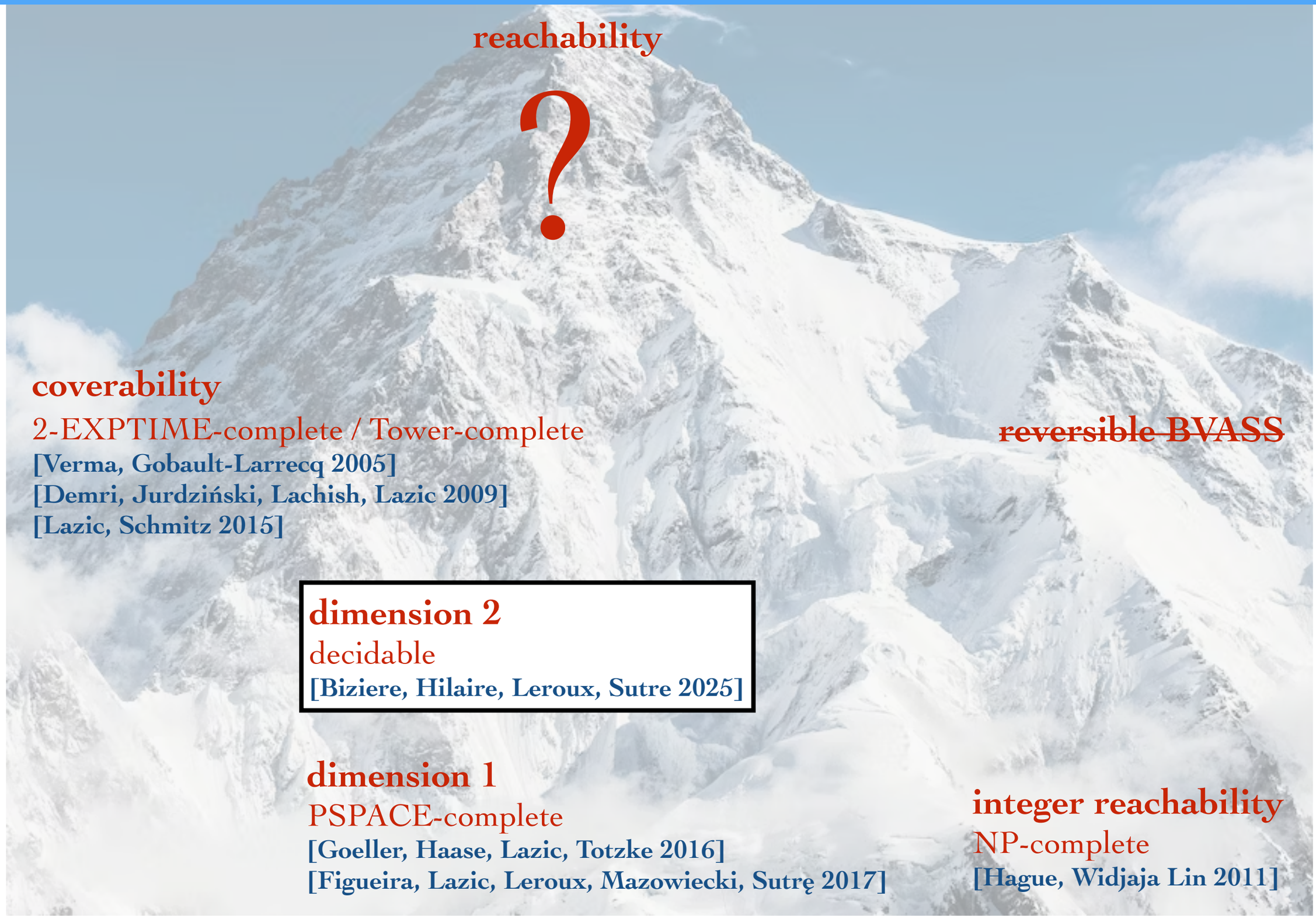
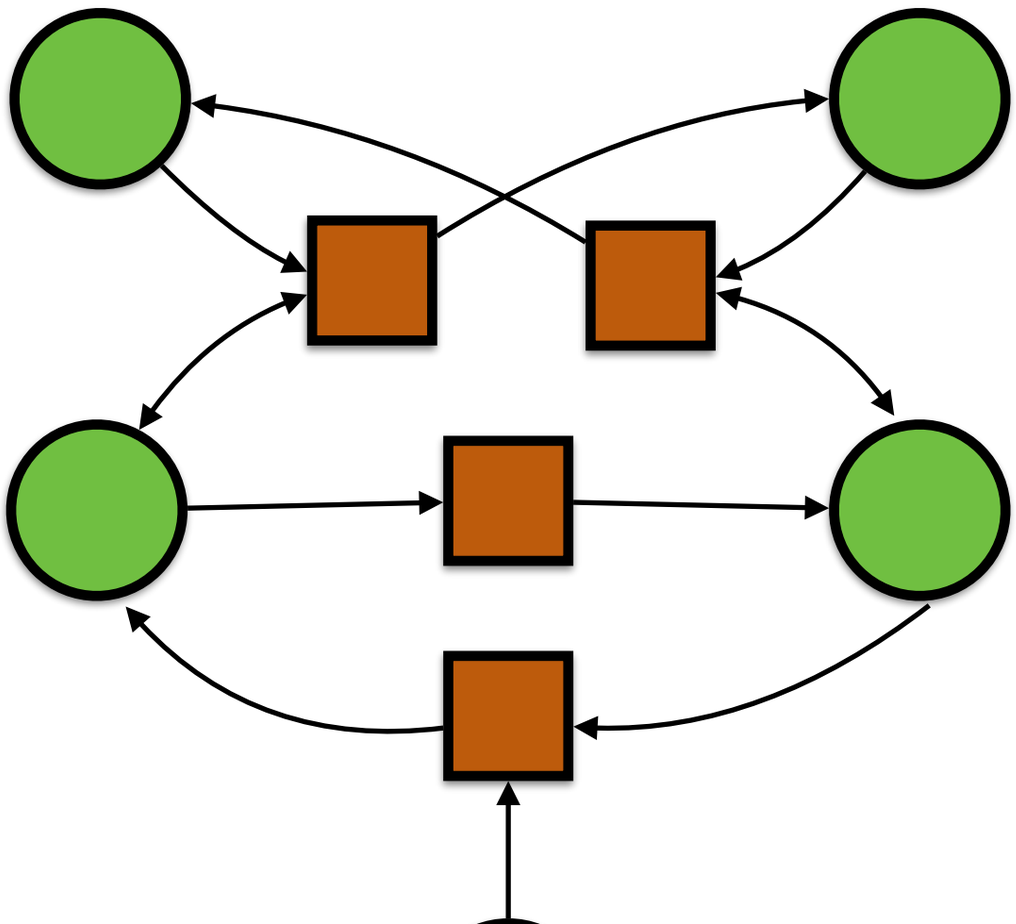
Branching VASS



Coloured PN



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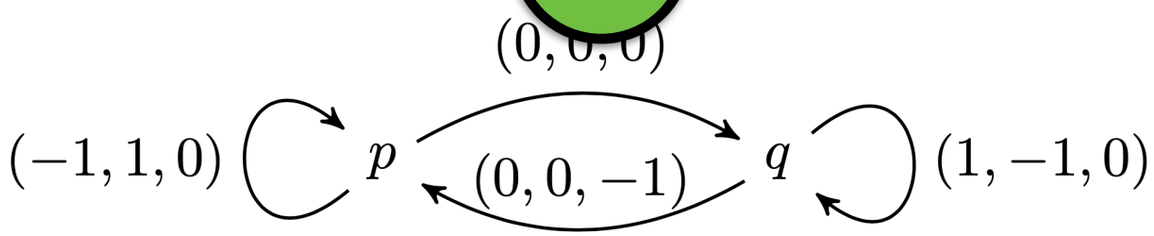

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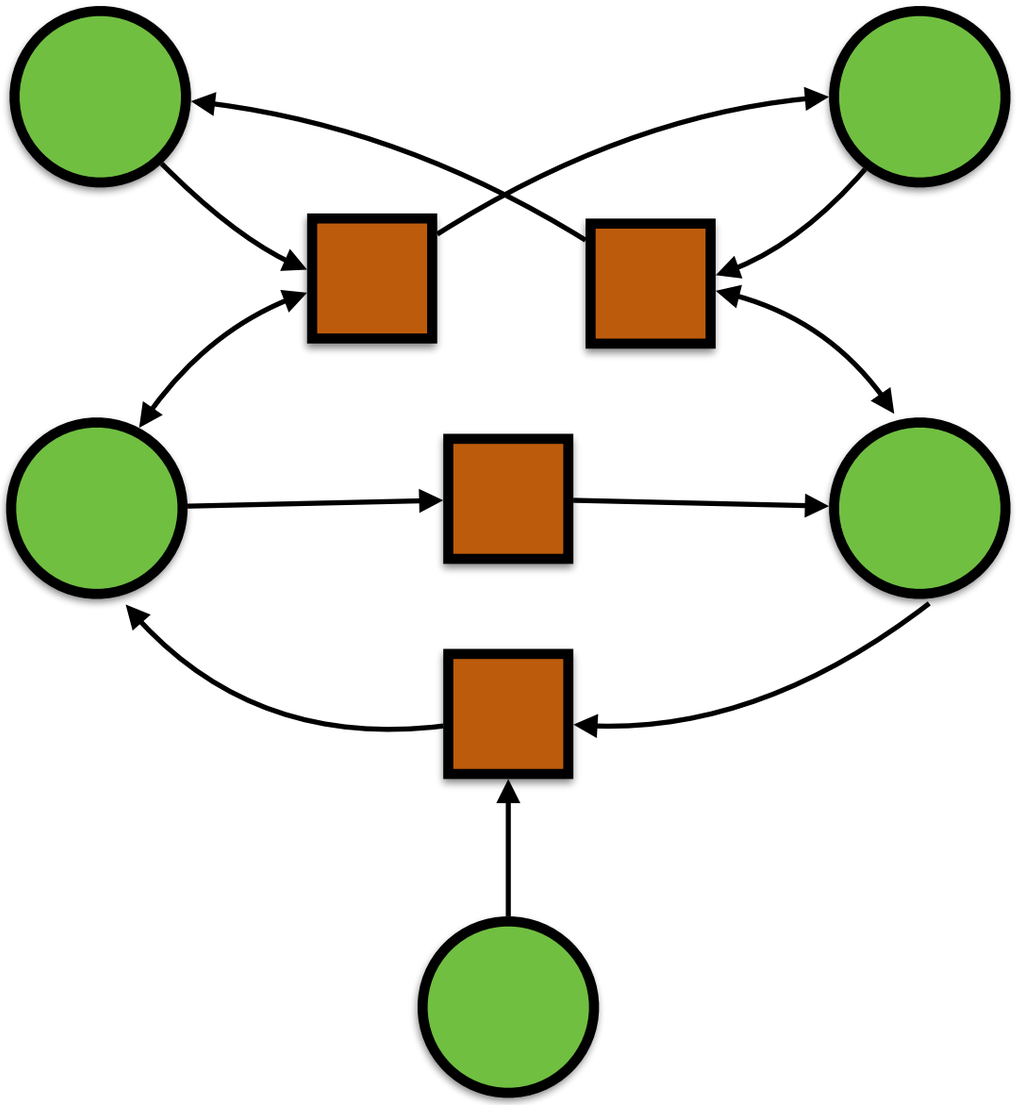

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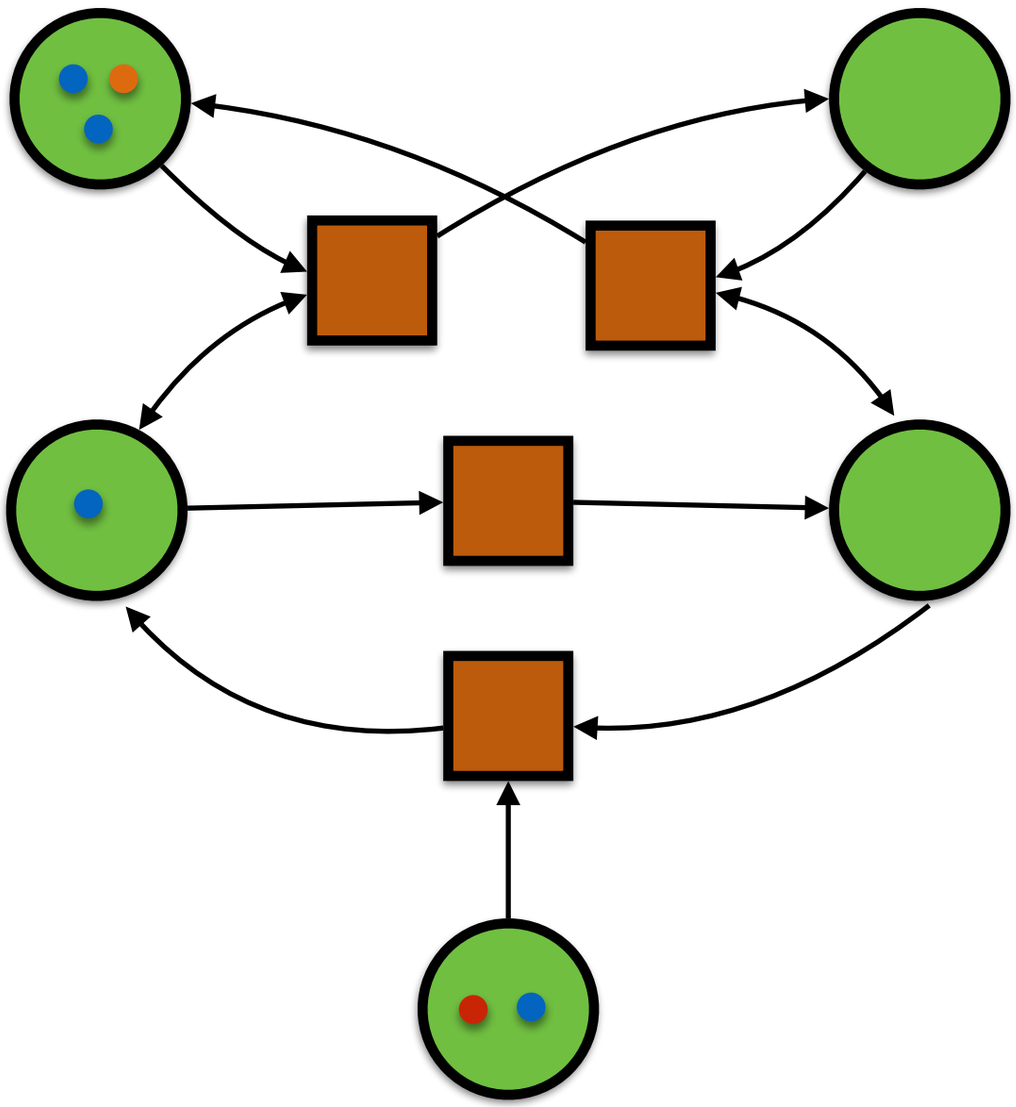

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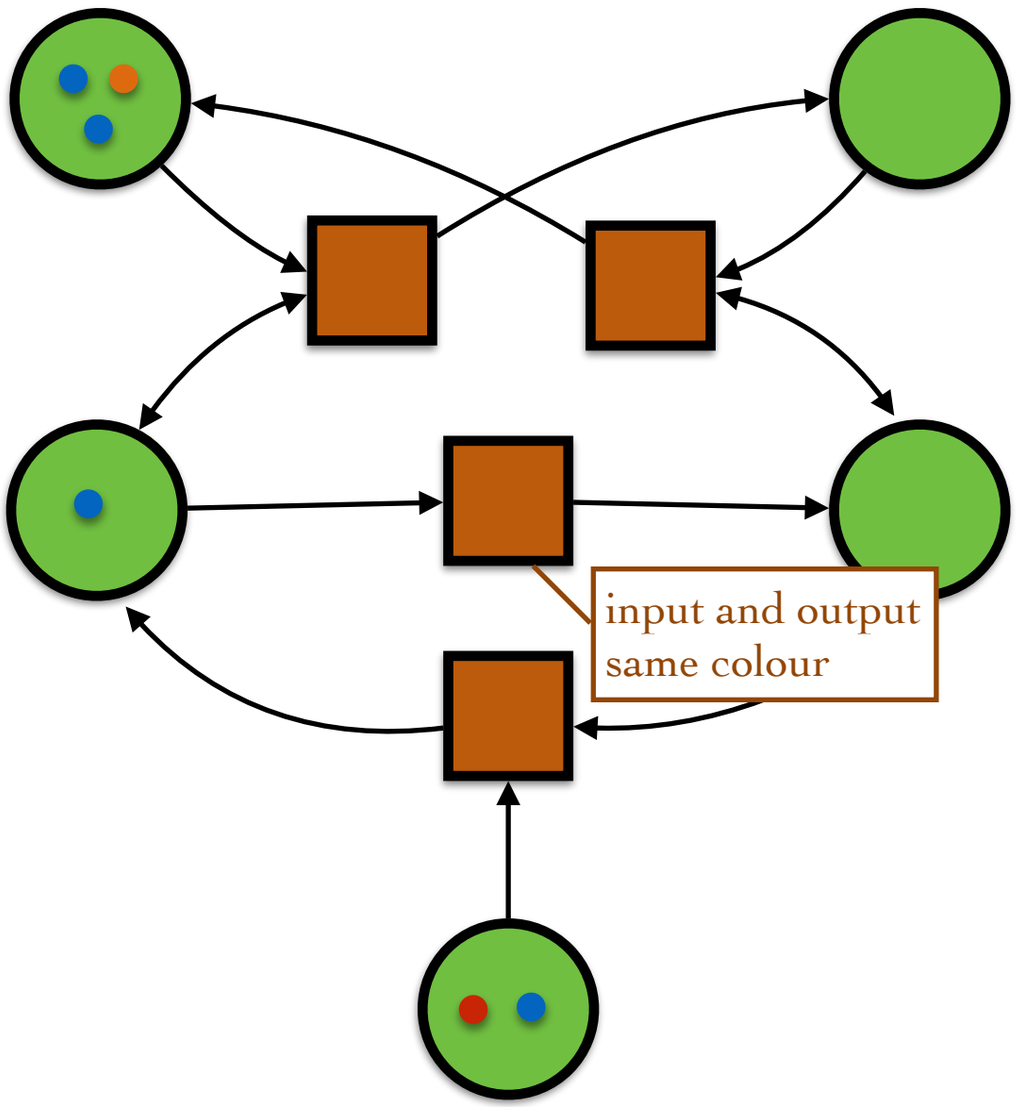

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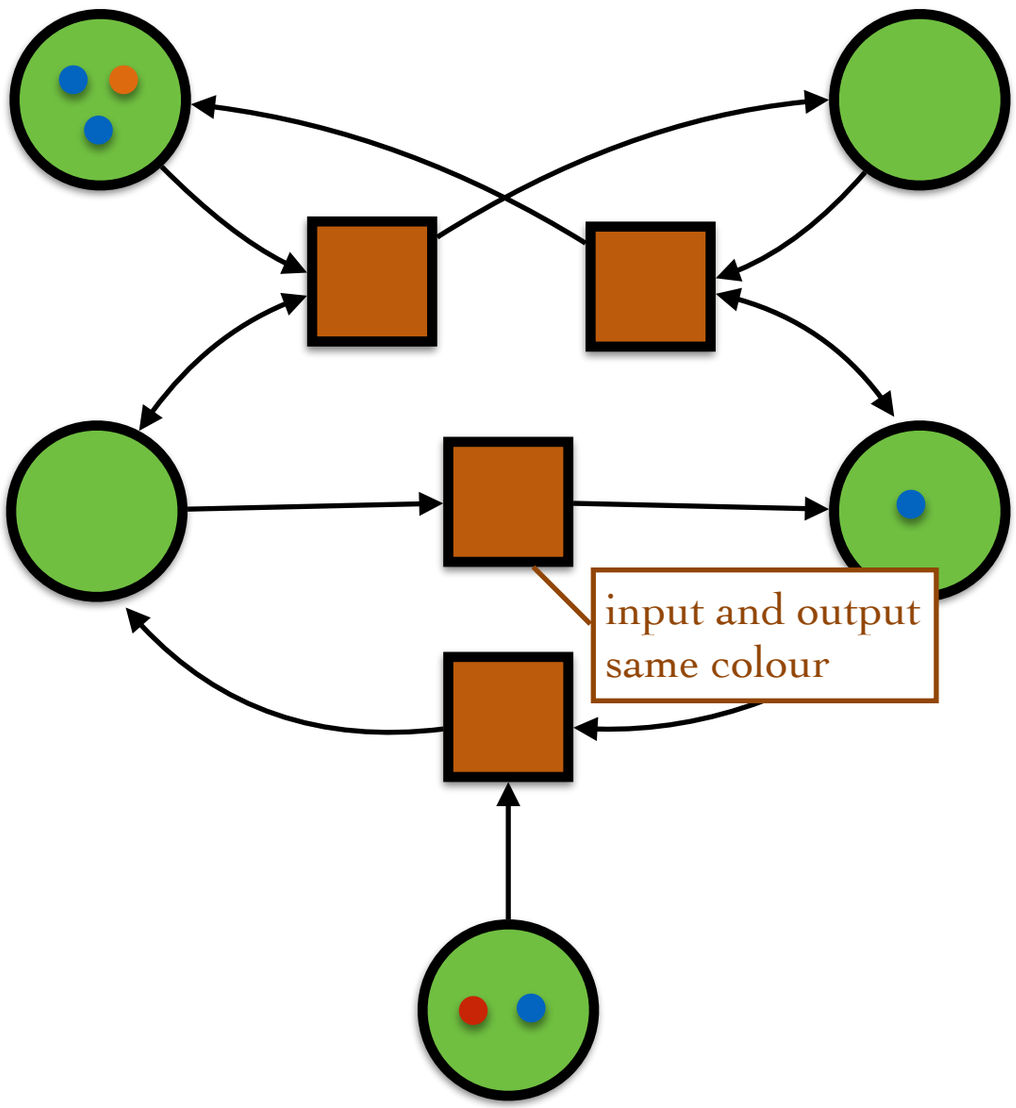

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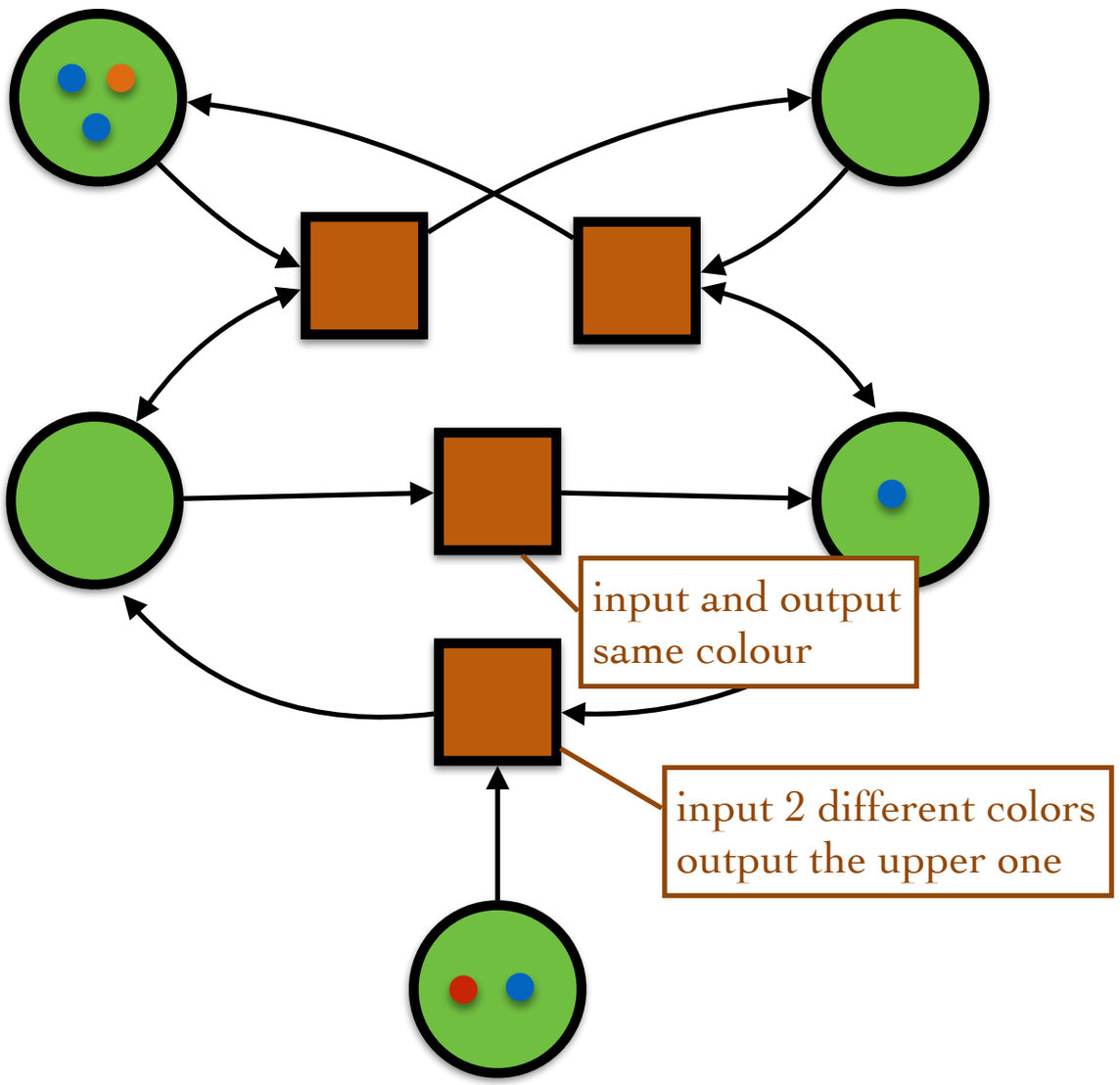
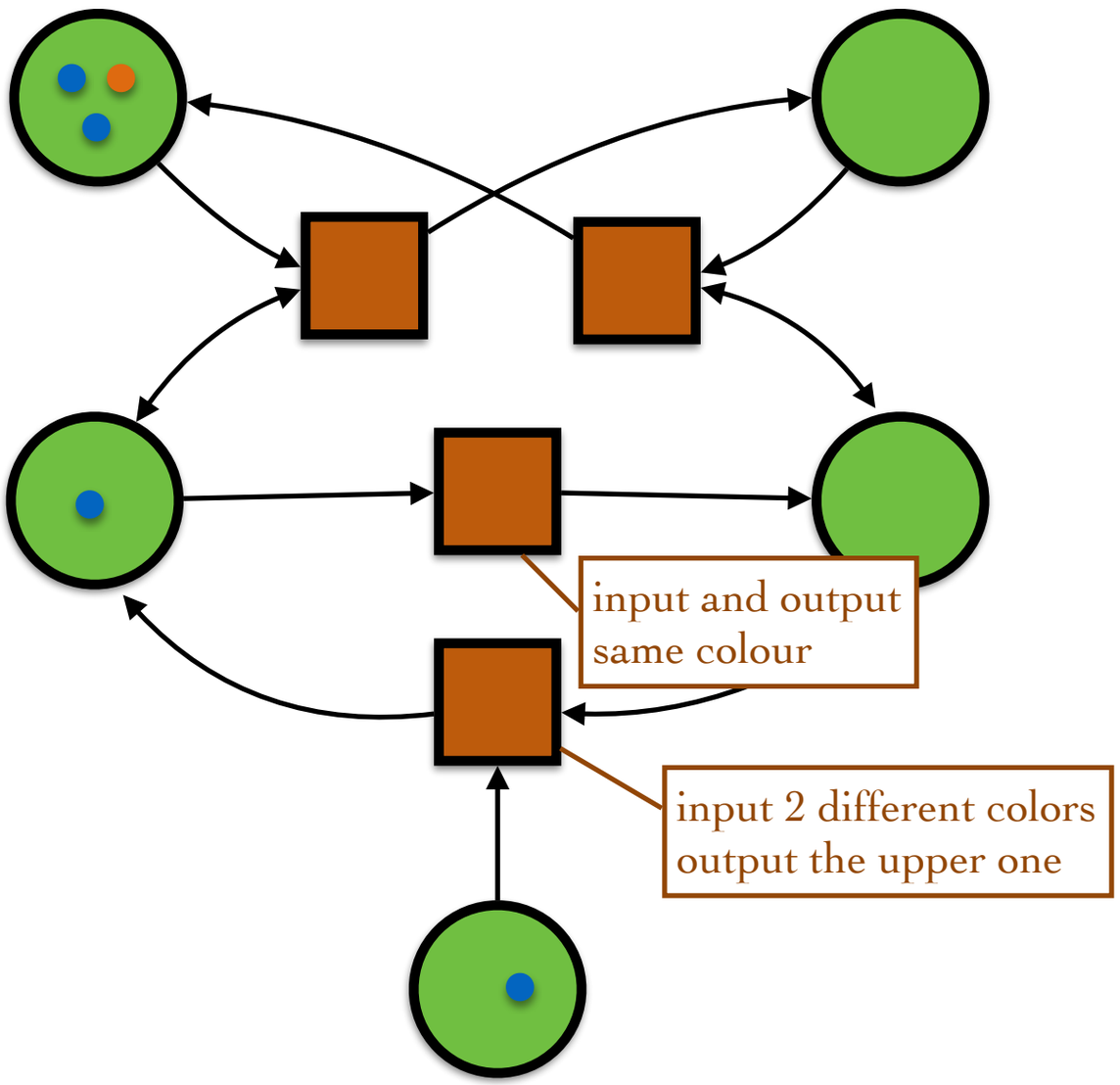

input and output
same colour
input 2 different colors
output the upper one

Coloured PN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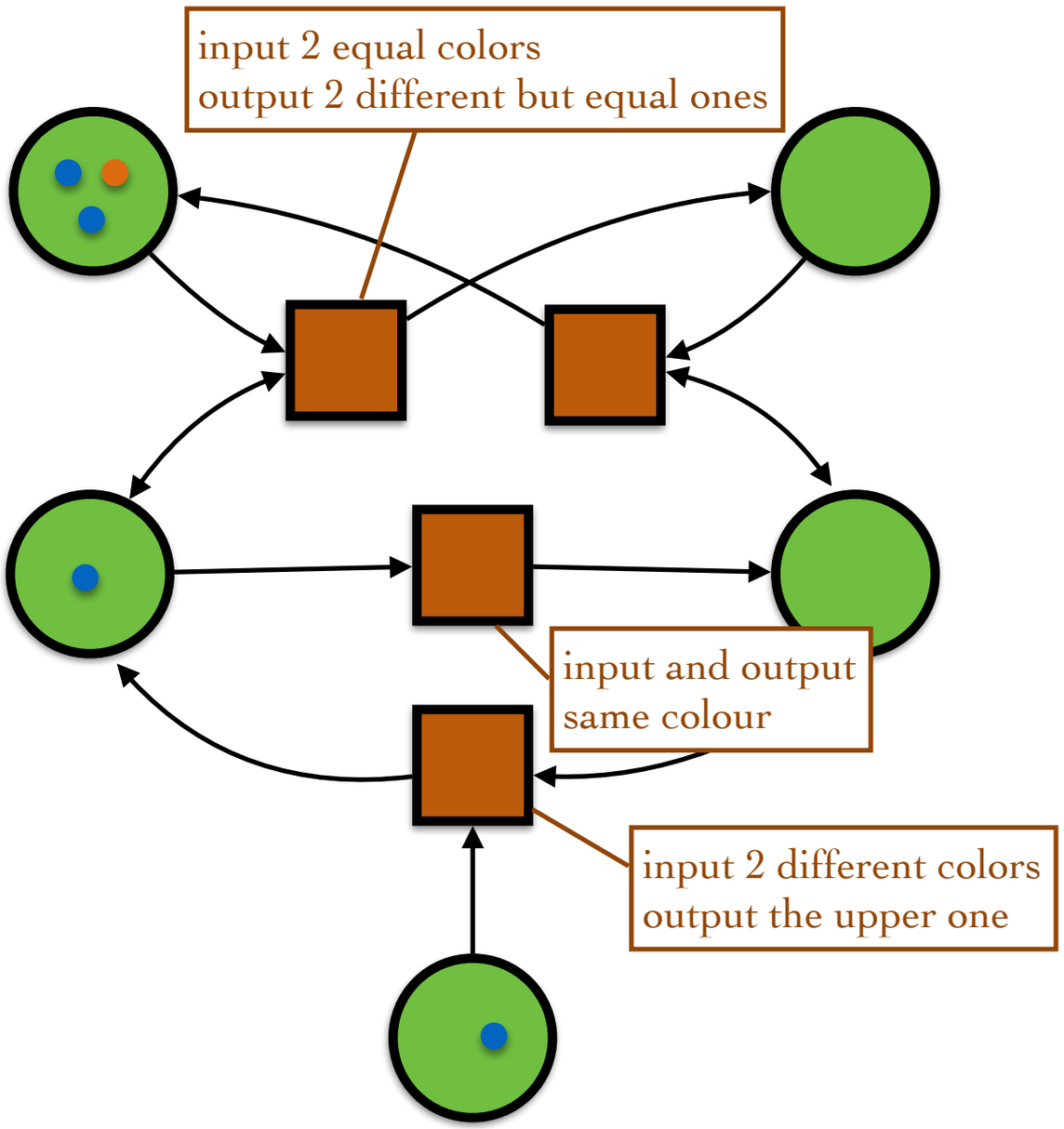
input 2 equal colors
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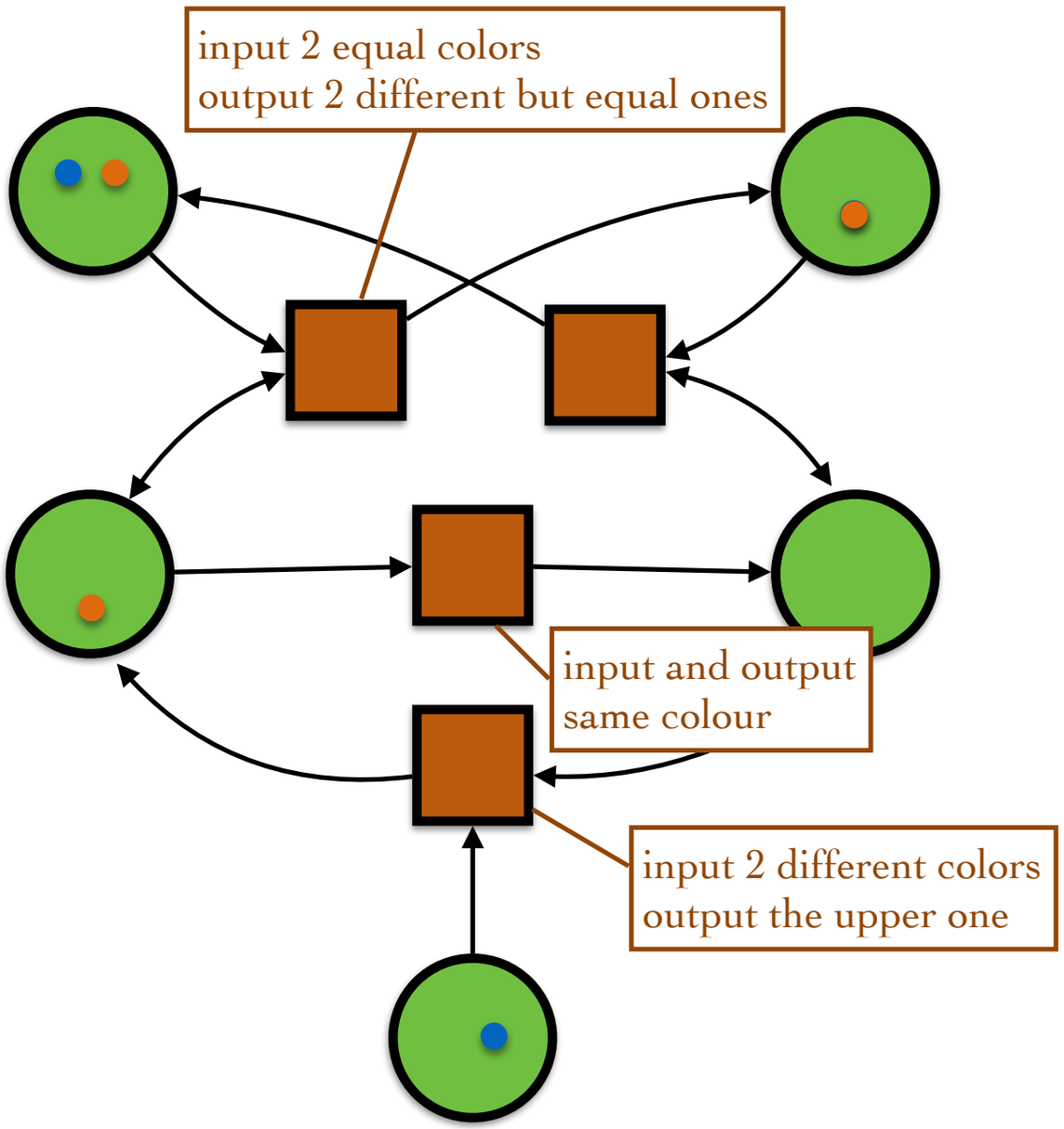
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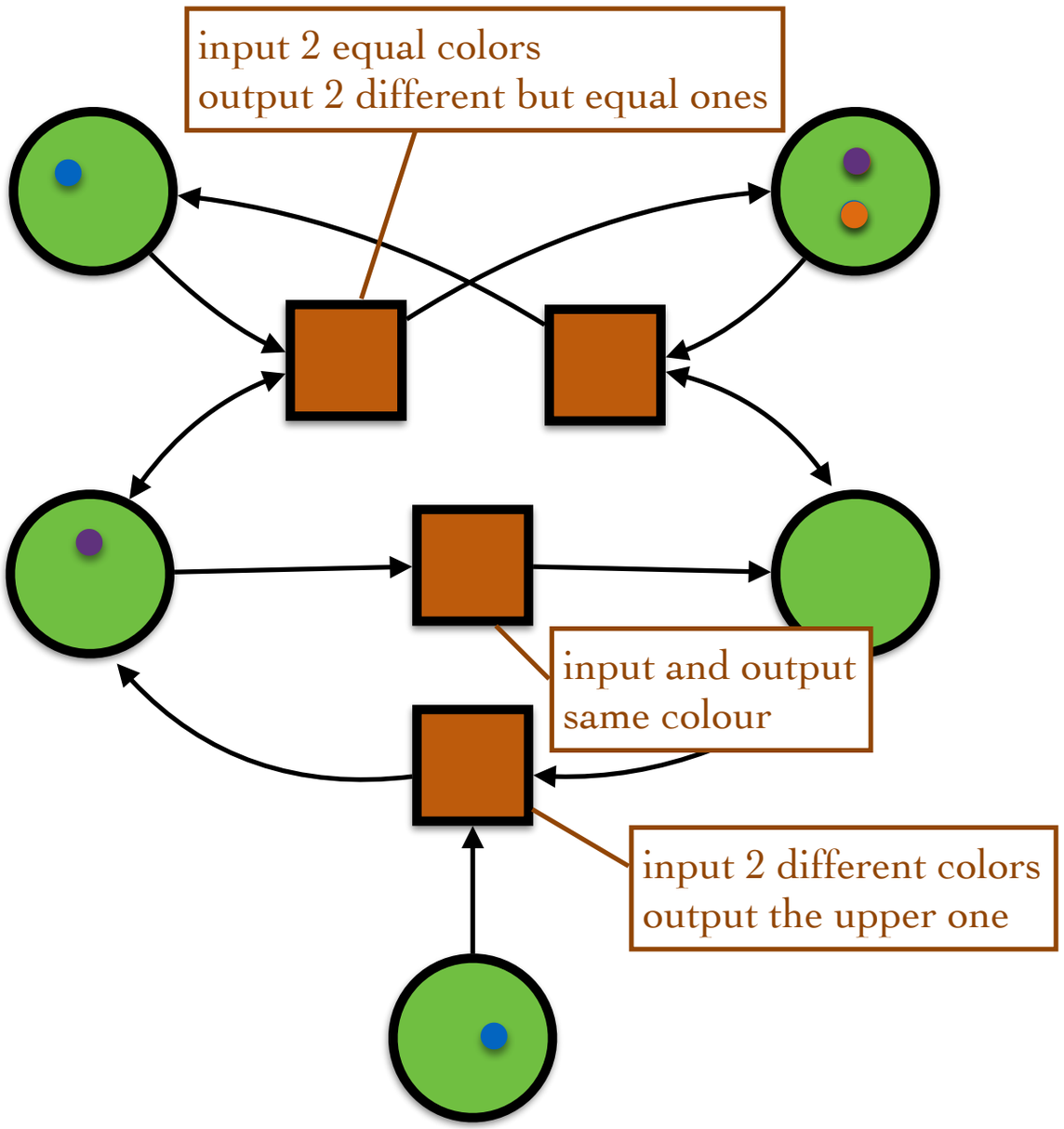
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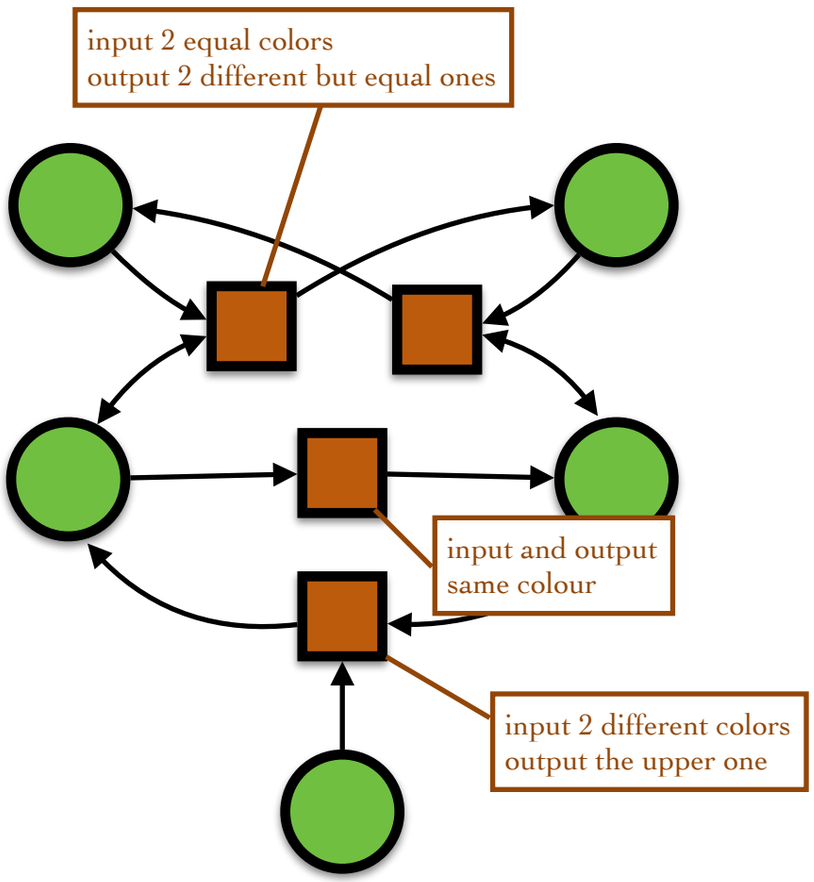
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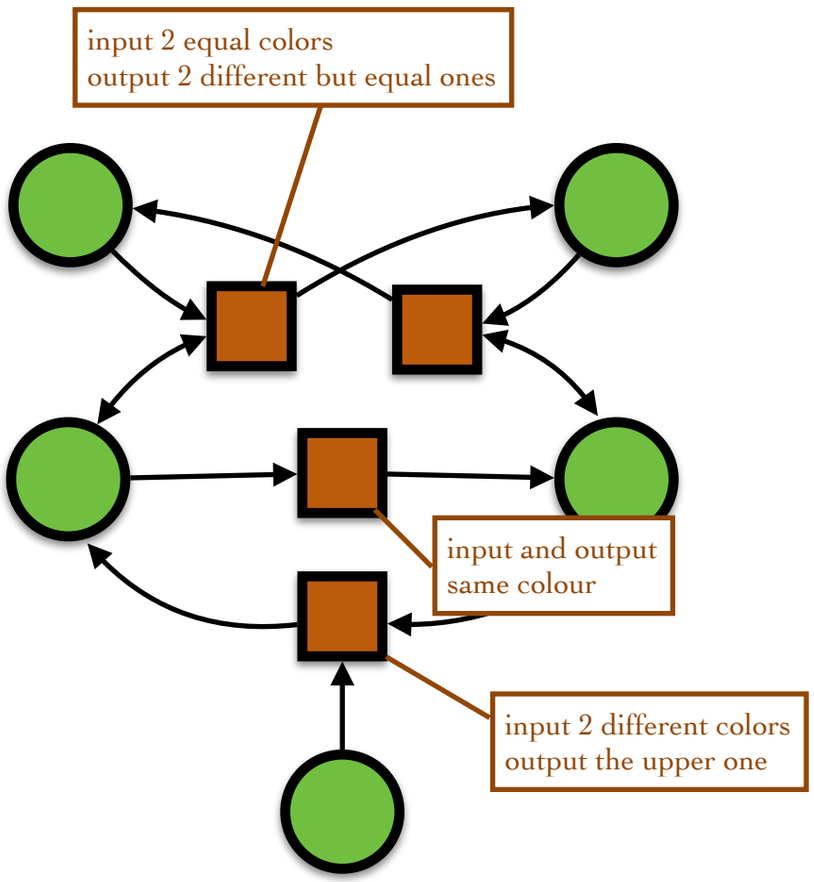

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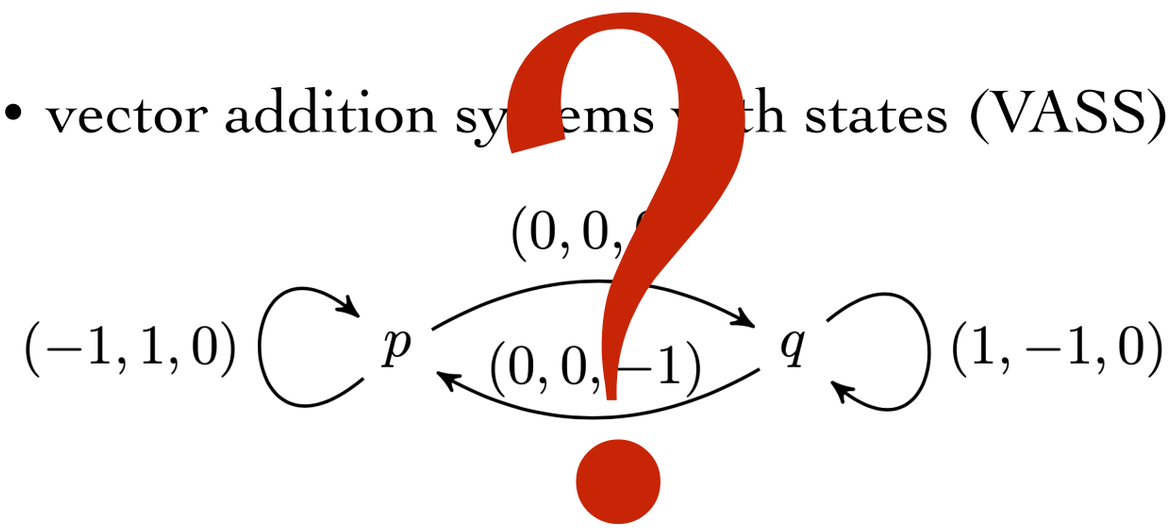

Coloured PN


reachability
coverability
$F_\omega \dots F_{\omega \cdot 2}$
[Lazic, Newcomb, Ouaknine, Roscoe, Worrell 2008]
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integer reachability
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[Hofman, Leroux, Totzke 2017]

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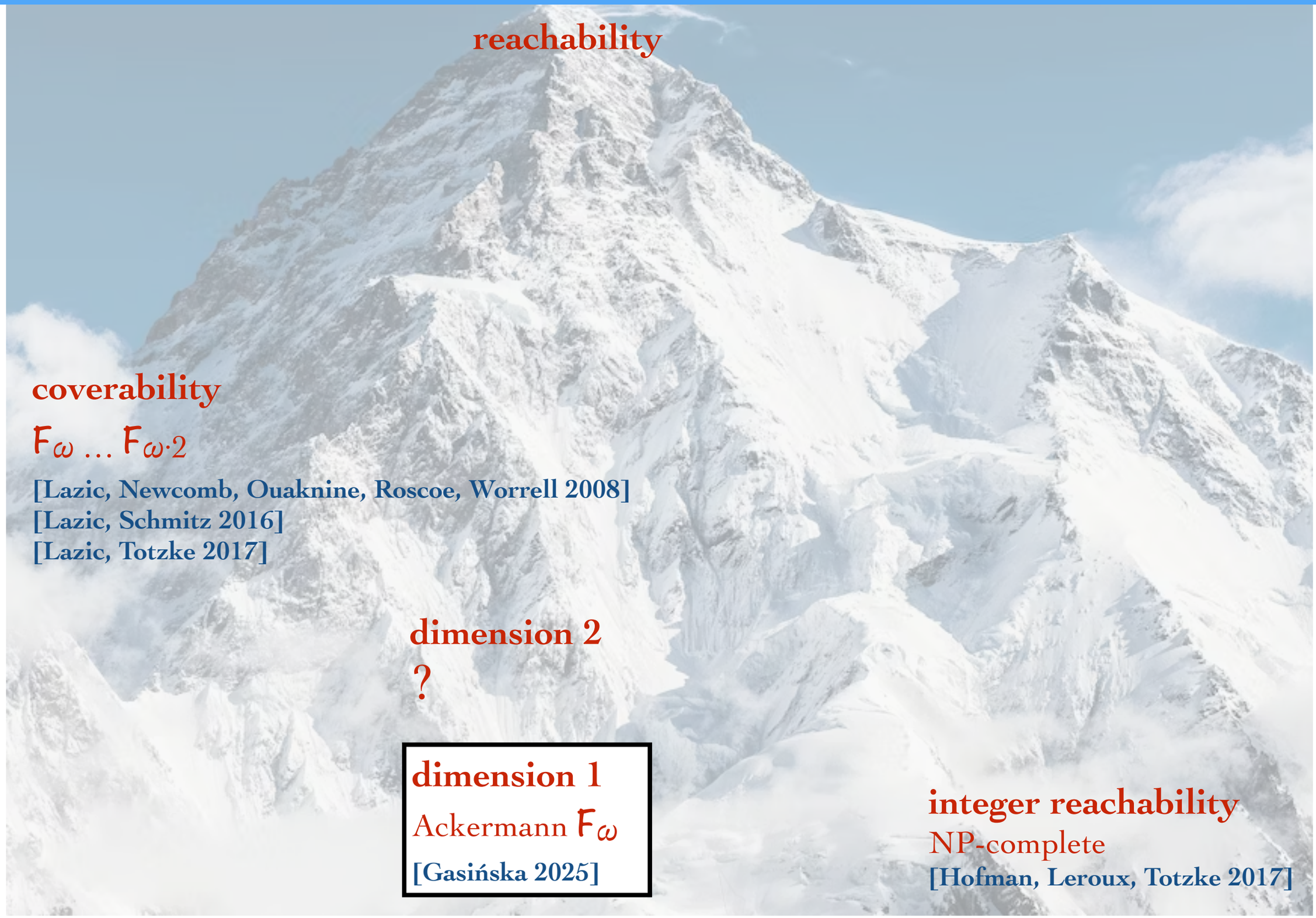
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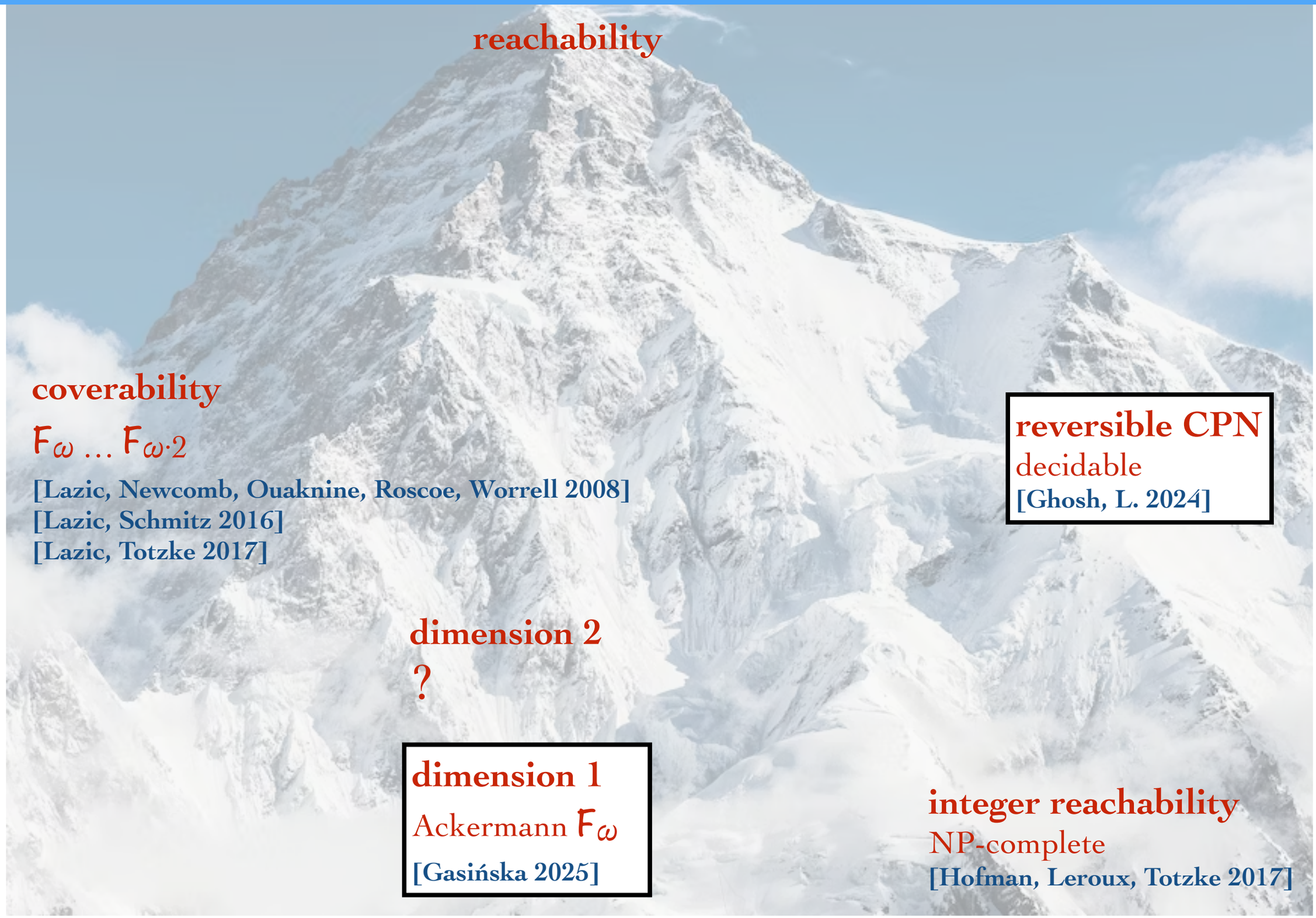
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Coloured PN


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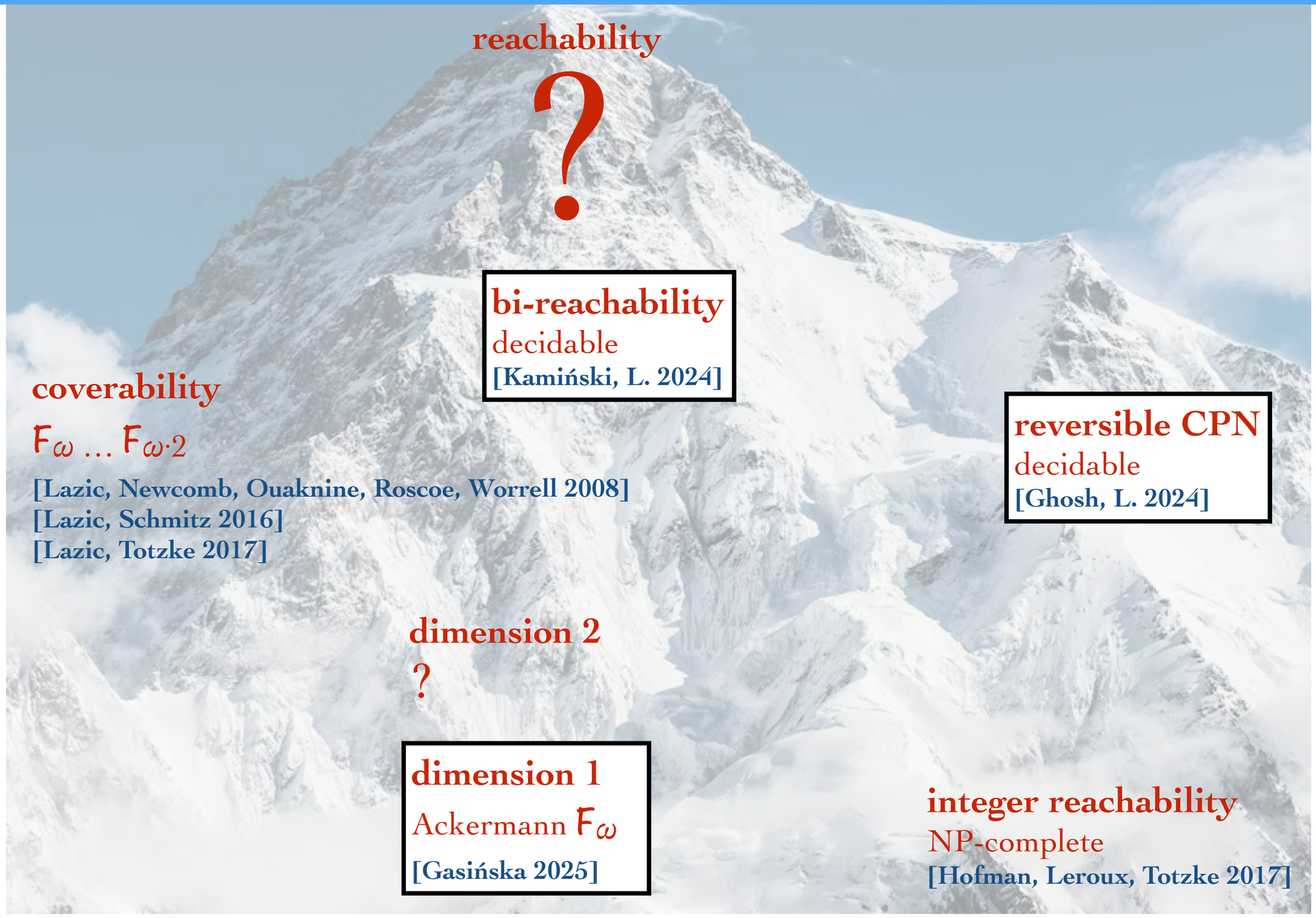
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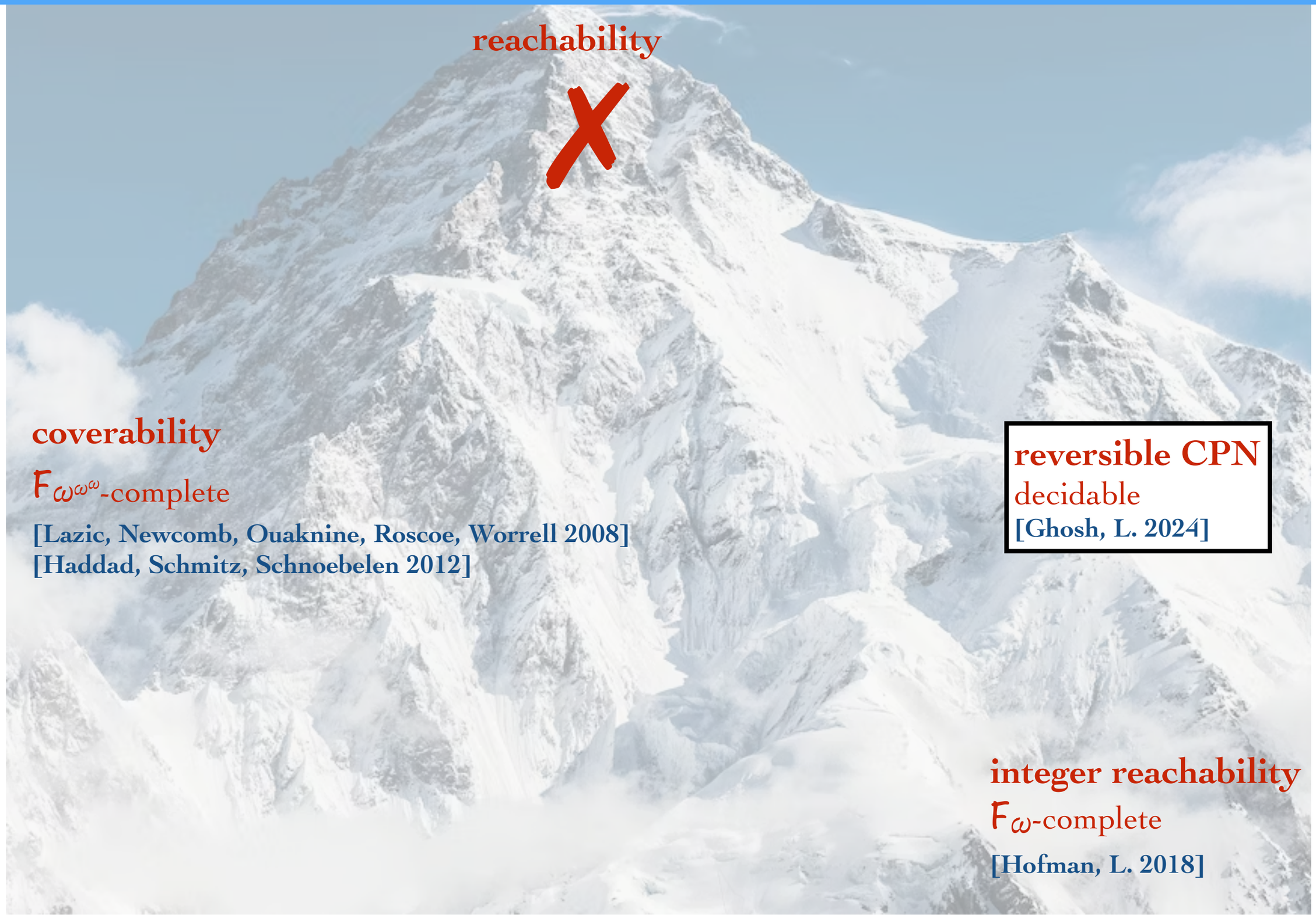
Coloured PN (ordered colours)



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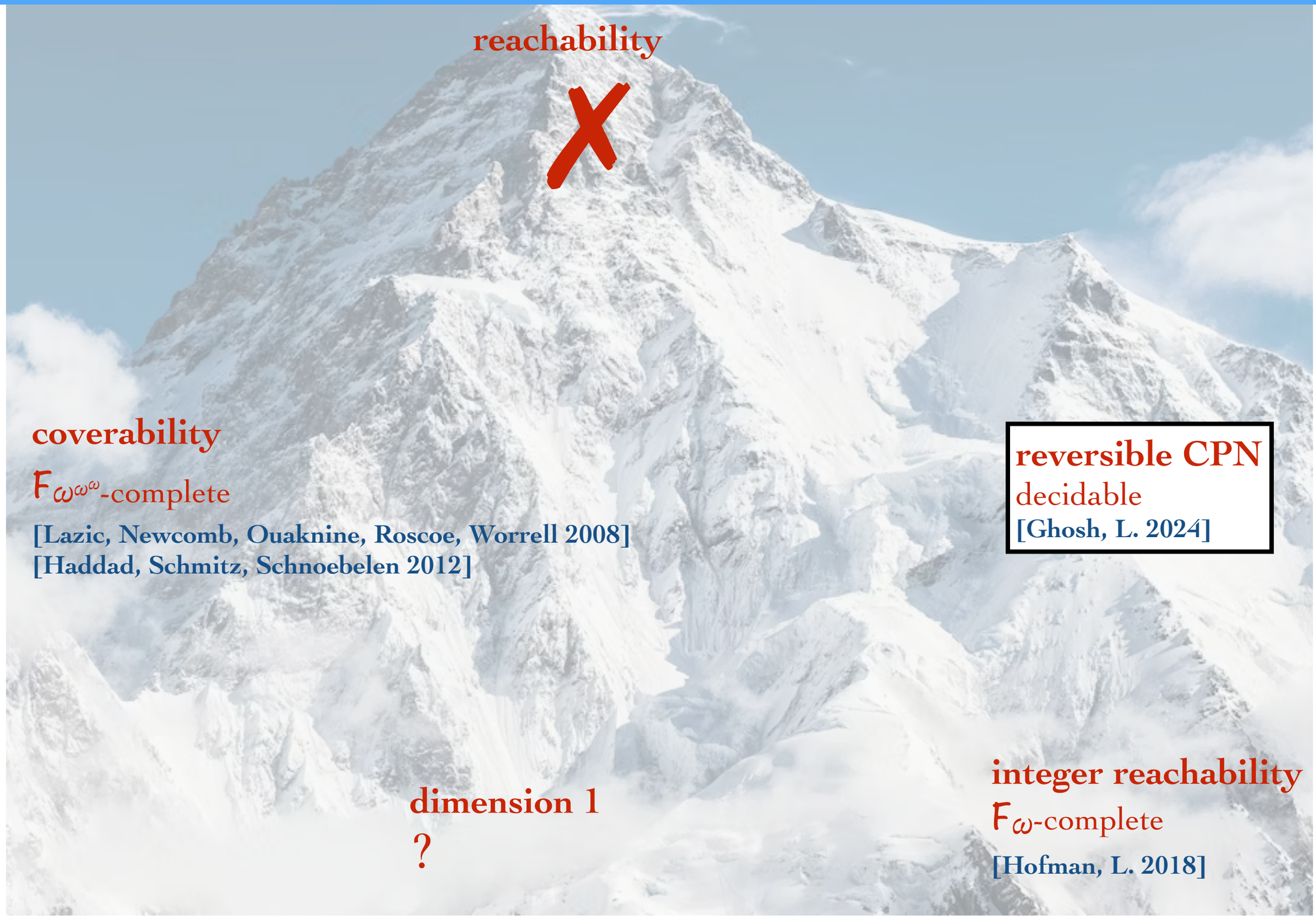

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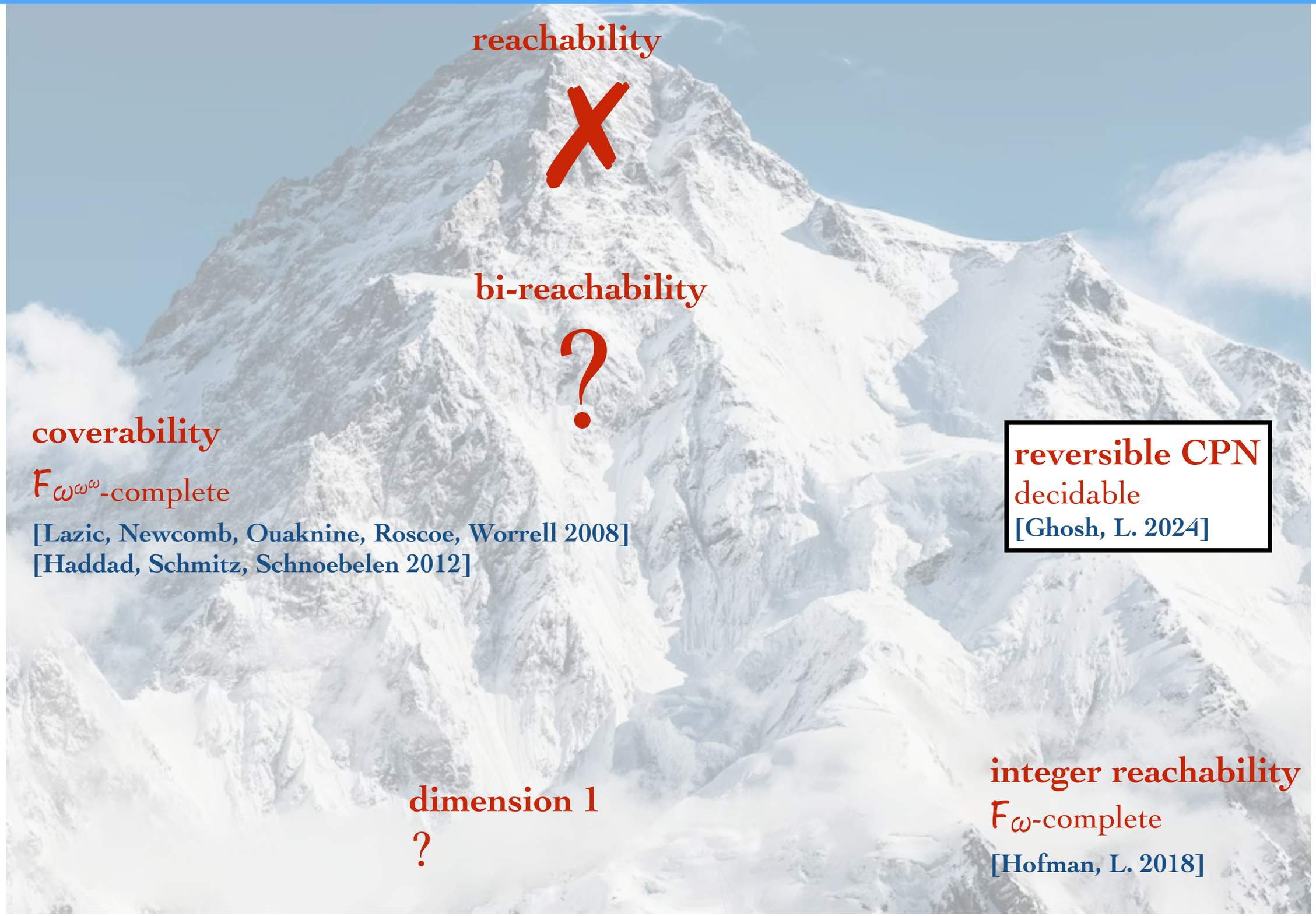
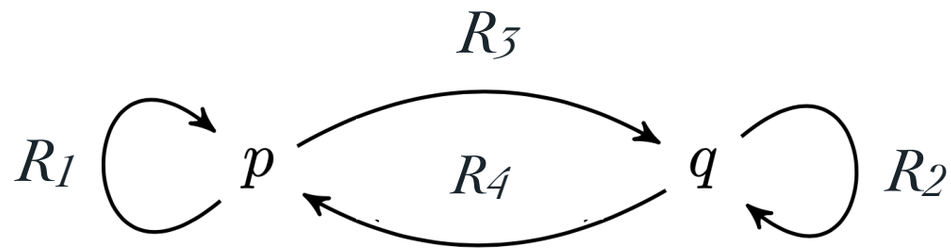


C-systems

$\mathcal{C} \subseteq \mathbb{N}^d \times \mathbb{N}^d$ step relations

admitting finite presentations



$R_1\ R_2\ R_3\ R_4\ \in \mathcal{C}$

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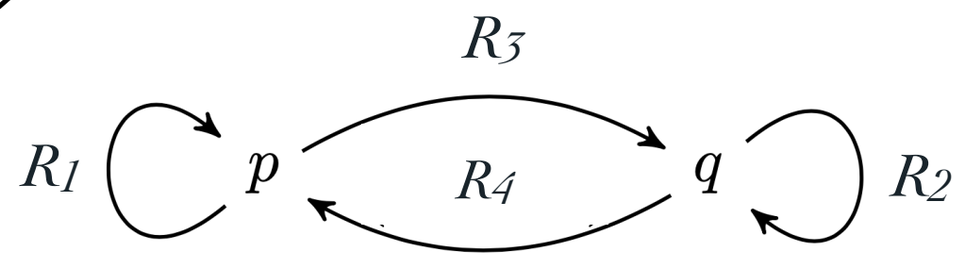

$R_1\ R_2\ R_3\ R_4\ \in \mathcal{C}$

C = Semilinear ?

C-systems

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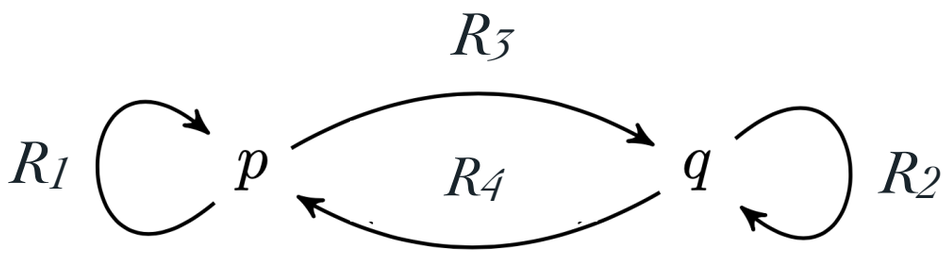
admitting finite presentations



$R_1\ R_2\ R_3\ R_4\ \in \mathcal{C}$

C = Semilinear ?

Semilinear-systems include counter machines !

Semilinear-systems are Turing-complete

monotone C-systems

$\mathcal{C} \subseteq \mathbb{N}^d \times \mathbb{N}^d$ step relations

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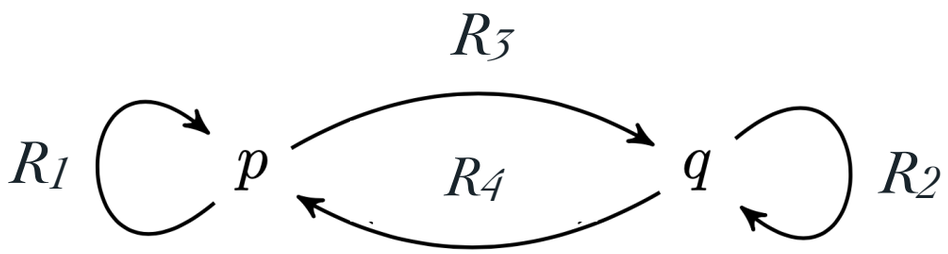

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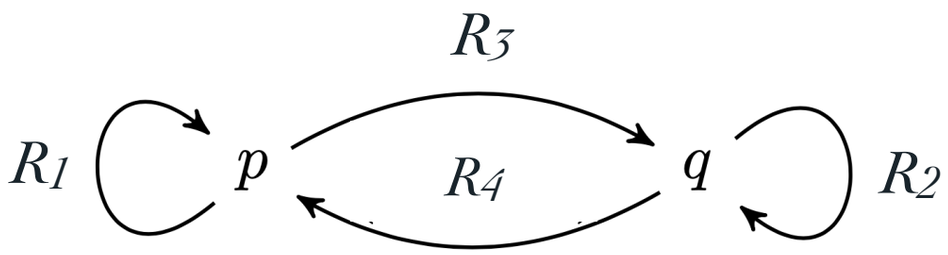

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and **monotone**

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$\Downarrow$

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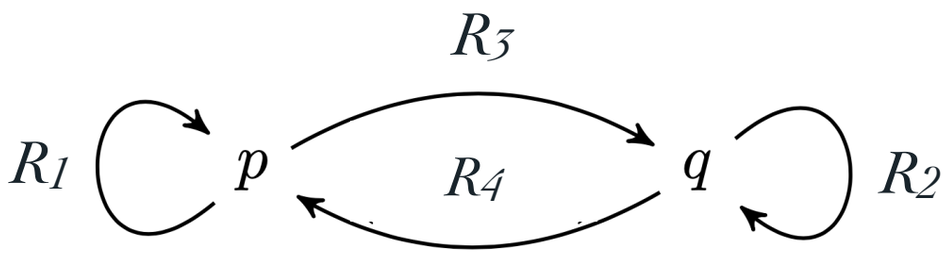

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[Czerwiński, Guttenberg, L. 2025]

monotone C-systems - applications

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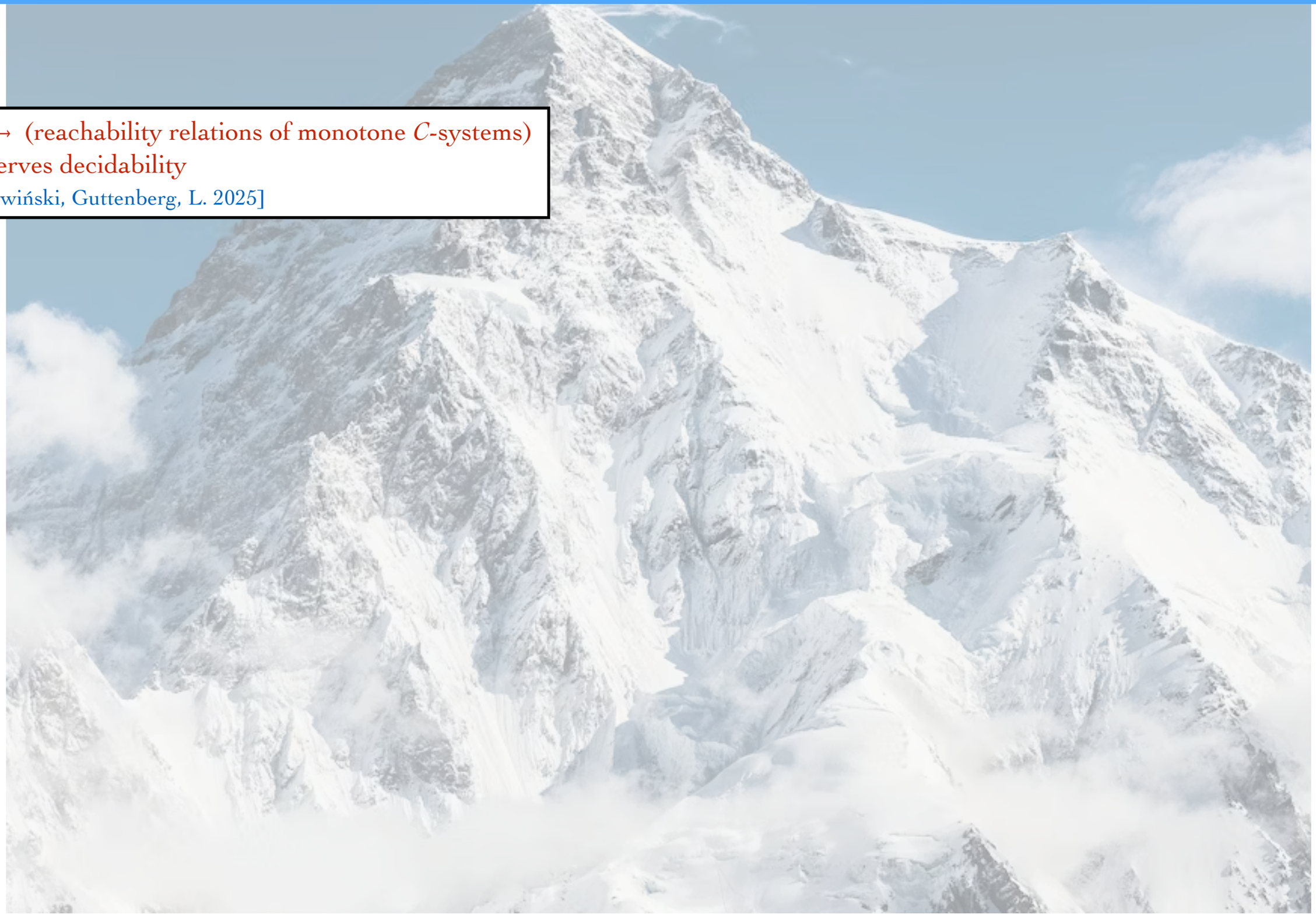

monotone C-systems - applications

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[Czerwiński, Guttenberg, L. 2025]

VASS with nested 0-tests

...

↑

VASS with 2 nested zero-tested counters

↑

VASS with 1 zero-tested counter

↑

VASS

↑

Semilinear

monotone C-systems - applications

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[Czerwiński, Guttenberg, L. 2025]

Pushdown VASS

…

CFG-controlled VASS of nesting 2

CFG-controlled VASS of nesting 1

CFG-controlled VASS of nesting 0

VASS with nested 0-tests

…

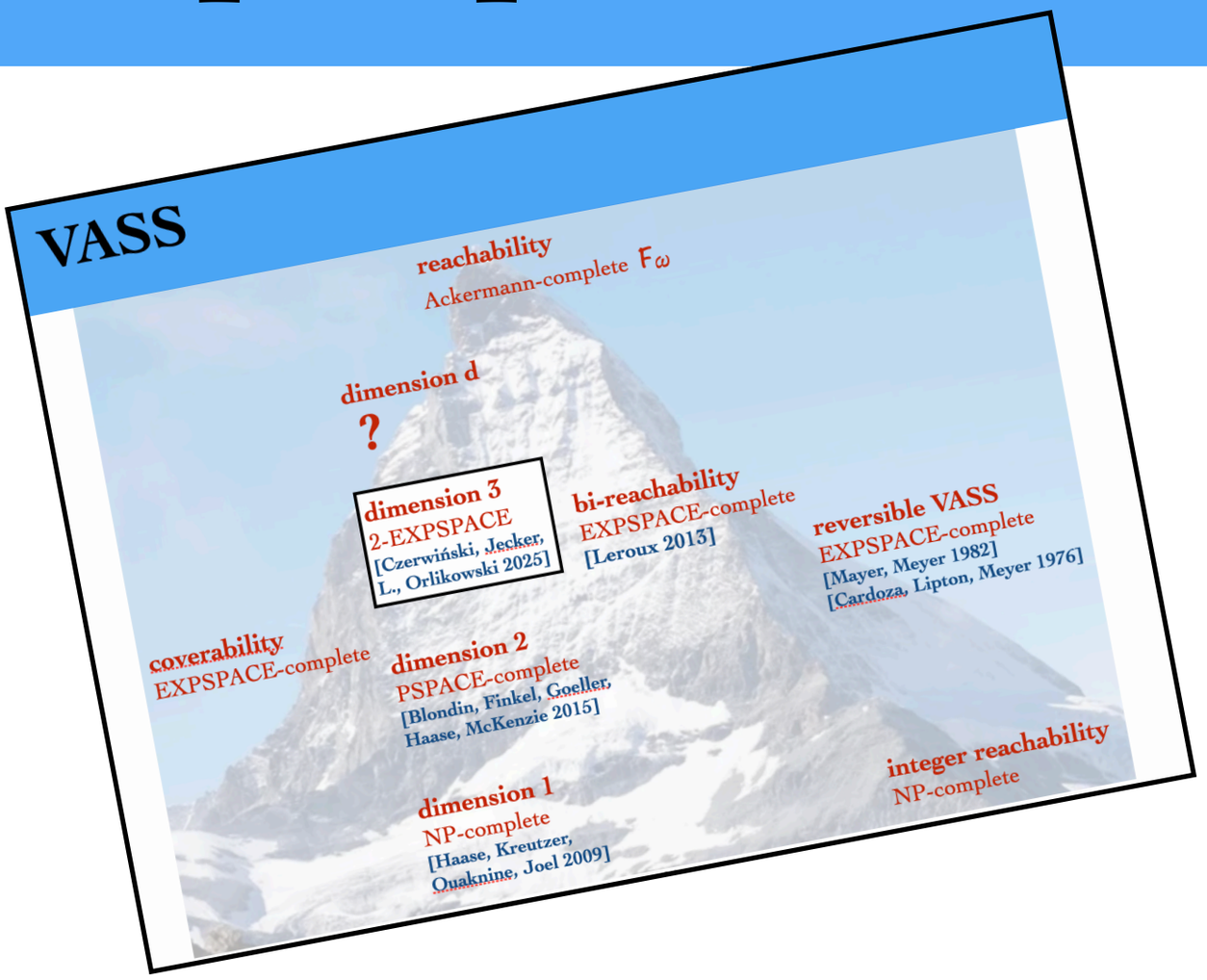
VASS with 2 nested zero-tested counters

VASS with 1 zero-tested counter

VASS

Semilinear

Open questions



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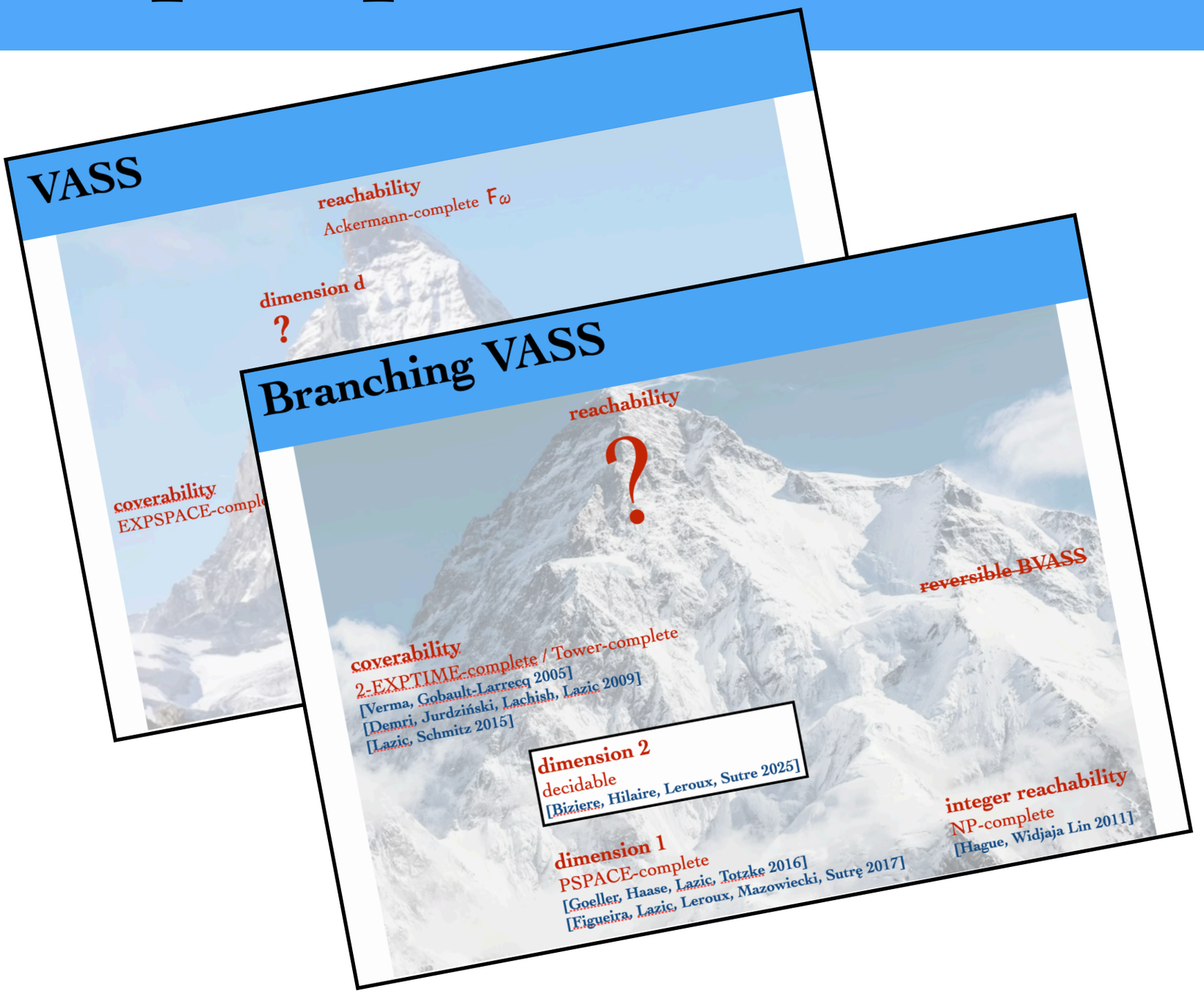

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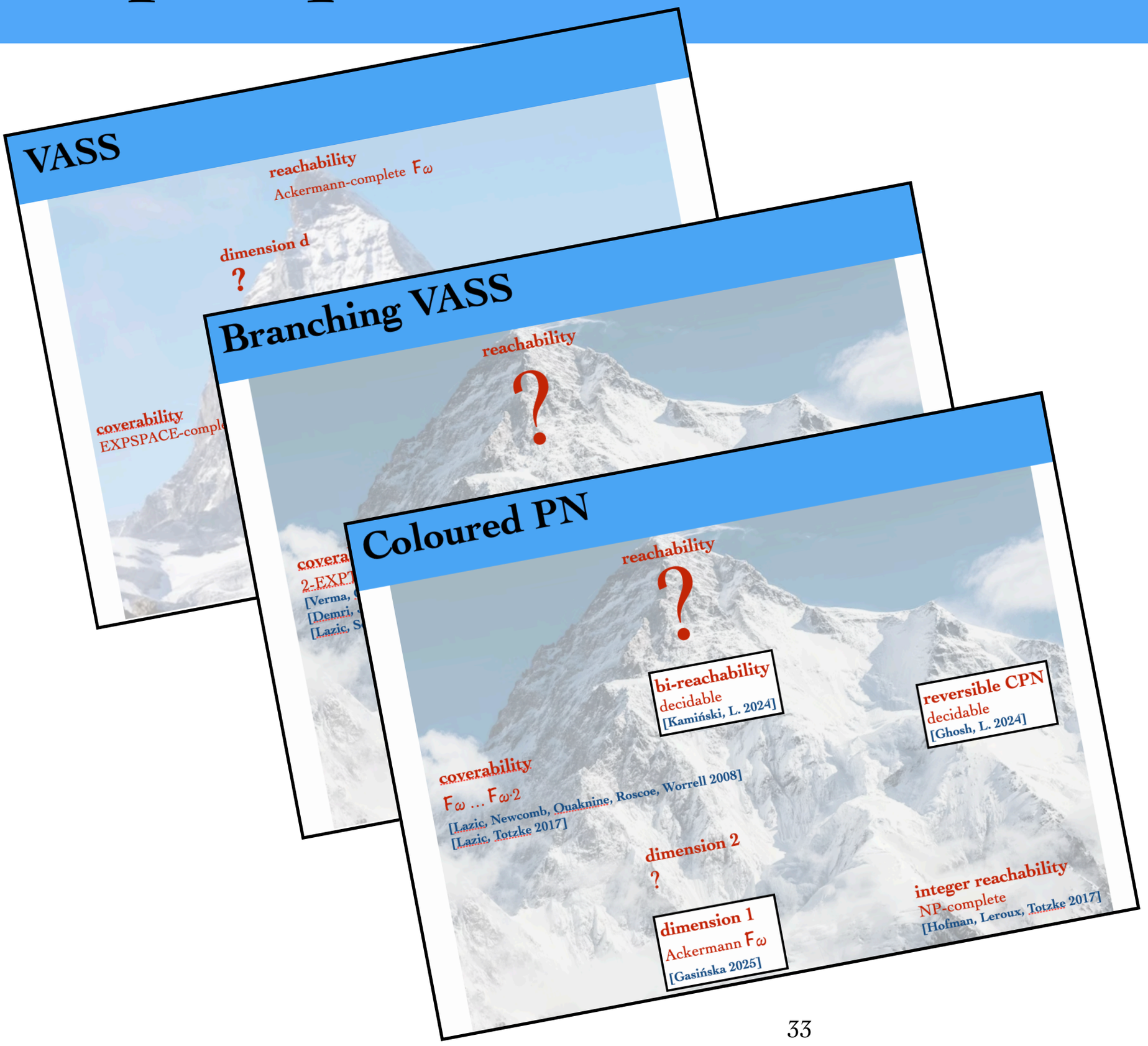

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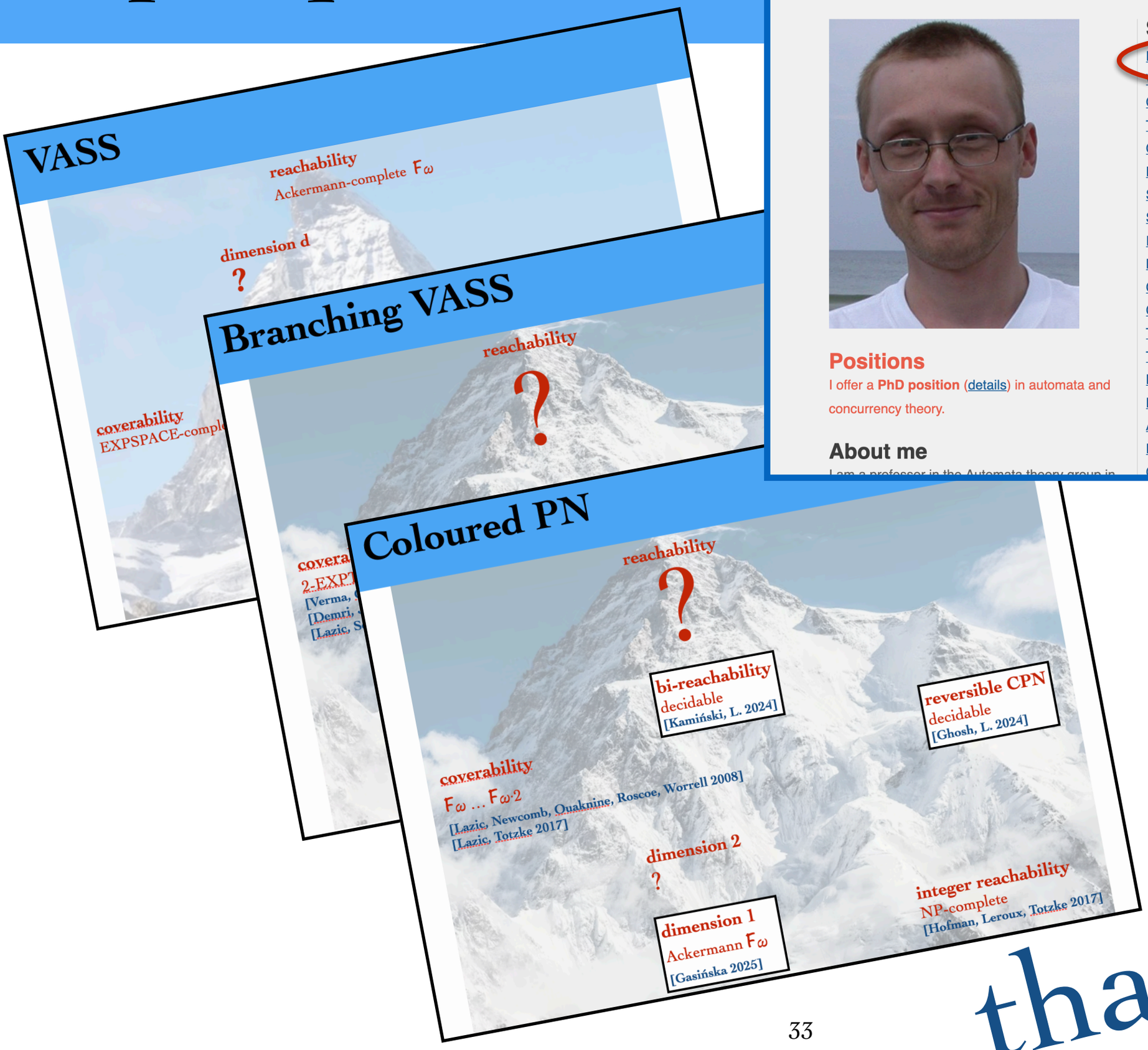

thank you!